This article discusses a workload allocation model in which tasks are matched to employees on the basis of a multi-dimensional skill measure. The main idea is to match positions and tasks to available and potential positions so as to minimize the differences in individuals' abilities and skill requirements. In addition to allocating existing personnel to positions, it is also possible to fire employees and hire new employees. The objectives of the mixed-integer optimization model include different types of costs and the proximity of an individual's capabilities to a task's ability requirements. A number of policies are formulated that allow different combinations of retraining of employees, as well as hiring and firing. These policies are applied to a real-life example that is solved by means of the constraint method. A variety of sensitivity analyses demonstrate the usefulness of the approach as a decision aid.

Keywords: staff assignment; workload allocation; ability space; job satisfaction; multi-objective optimization

1. Introduction

The purpose of this article is to provide a model that attempts to assign jobs to employees, so as to minimize costs and to ensure that the allocation respects the employees’ abilities and fosters job satisfaction. In addition, this article allows management to (re-)train employees, hire new employees and fire existing employees. These courses of action will be chosen so as to minimize costs, use the employees’ capabilities as efficiently as possibly and distribute the workload equitably among the employees. The model presented here is an extension of work done earlier by Eiselt and Marianov (2008). The authors previously presented this model as a first step towards a comprehensive model for workload assignments. As opposed to the model presented in this article, it includes cost, distance and equity objectives, but does not allow hiring and firing, i.e. it works only with existing employees. In fact, the main focus of this article is an investigation of the effects of retraining, hiring and firing with respect to the benefit of the employees. For that purpose, this model evaluates a variety of strategies that make different uses of these management options. In contrast to the well-studied field of rostering, scheduling issues are not included in the model in this article.
As was the case in the aforementioned study by the present authors, the foundation of this research can be found in earlier work regarding job satisfaction. The main idea behind making efficient use of employees’ abilities is to match task requirements and employees’ skills closely to avoid boredom which, as most authors agree, results in absenteeism, voluntary termination, early retirement or other forms of withdrawal from work.

Early studies into job satisfaction include the works of Herzberg and co-workers (1959, 1966). Their two-factor theory distinguishes between job content factors (motivators), the presence of which is an incentive or motivation to do the work (better), but the absence of which is not necessarily perceived as negative. In contrast, there are job context factors (hygiene factors), whose presence does not provide any added motivation to do the job, but whose absence results in dissatisfaction. Many subsequent authors have noted a correlation between job satisfaction and various types of withdrawal, e.g. absenteeism or high staff turnover. Prominent examples of studies that come to this conclusion are Vroom (1964), from the job enrichment school of thought, who posits that different job features imply different degrees of job satisfaction, Talacchi (1960), Atchison and Lefferts (1972), Locke (1976), Scott and Taylor (1985), Petterson et al. (1995) and Sagie (1998). Waters and Roach (1979) found a significant correlation between job satisfaction and job termination within the first year of employment. Roessler et al. (2004) determines factors of job (dis-)satisfaction among employees with multiple sclerosis, while Perry et al. (2000) investigated job satisfaction among college graduates. The latter two studies found significant correlation between job dissatisfaction and absenteeism.

The link between job satisfaction and skill mismatches was pointed out by Cabral Vieira (2005), who indicated that mismatching the skills of an employee and the skill requirement of a job is detrimental to job satisfaction (this result is key to the model presented in this work). The Workopolis website (Workopolis 2007) names the opportunity to use acquired skills as one of the key factors in job satisfaction. Sutherland and Cooper (2000) made the same inference regarding jobs that require few skills, leading to boredom, low job satisfaction and resulting staff turnover. In contrast, Bassett (1994) made the case that some workers actually prefer mindless repetition in their jobs. However, some employees, especially those with higher education, resent narrow specialization in their jobs and react with withdrawal. On the other hand, Kilbridge (1961) saw no relation between repetitiveness and job satisfaction, assuming that job satisfaction is measured by factors such as voluntary termination and absenteeism.

Measuring job satisfaction is another important issue. Early efforts include the Minnesota satisfaction questionnaire by Weiss et al. (1967) and the job descriptive index by Smith et al. (1969) (for an updated user manual, see Balzer et al. (1990)). A more recent measure is the job satisfaction survey by Spector (1997). Stahl and Luczak (2000) elaborated on the difficulties of transforming qualitative evaluations into quantitative measures. They used fuzzy membership functions to deal with abilities, defined as traits that are inherent in people and do not change much with education and learning (as opposed to knowledge and skills that can be learned by training). Another important strand of research was prompted by Seal (1945) and later Young and Almond (1961) and Bartholomew (1963). These authors discussed manpower planning models in hierarchies by modelling the structure of the hierarchy as a Markov model. A classical survey is the book by Vajda (1978), while a more recent treatment of the subject is provided by Vassiliou (1998). Similarly to this article, Georgiou and Tsantas (2002) included not only hiring and firing in their model, but also allowed for the training of existing personnel. However, all Markov models address a level in a hierarchy as the smallest unit, whereas the model in this article deals with individuals as the basic unit.

The remainder of the article is organized as follows. Section 2 presents the structure and discussion of the model as well as a formal multi-objective mixed-integer, linear programming formulation. Section 3 discusses the computational results of a real-world application. Section 4 summarizes the findings and provides an outlook for future research.
2. The model

The structure of the model includes three major components—employees, positions in a multi-dimensional ability space and tasks. While the ultimate idea is to assign employees to tasks, this assignment is accomplished by way of the abilities that, on the one hand, are possessed by the (new or retrained existing) employees and, on the other hand, are required by the tasks at hand. The tripartite graph in Figure 1 demonstrates this basic relationship.

First consider the employees. The set of employees $K$ is subdivided into two mutually exclusive and collectively exhaustive subsets $K_e$ and $K_p$, where $K_e$ denotes the set of existing employees (i.e. the set of employees presently on the payroll of the organization under investigation) and set $K_p$ denotes the set of potential new employees. As far as the new employees are concerned, they may either be taken from a pool of known individuals who have applied for jobs in the organization, or taken from a ‘wish list’ that includes a set of ‘ideal candidates’ the organization is looking for.

The abilities of the employees are mapped in a multi-dimensional ability space (as opposed to the single-dimensional skill levels used by authors such as Eitzen et al. (2004)), in which each dimension represents a particular skill. Once an employee’s abilities have been measured, that individual can be mapped into that space as a point. Similarly, each task requires certain levels of different skills. Again, once they have been determined, each task can also be mapped as a point in the ability space. The assignment of tasks to employees can then be seen as a matching of two sets of points in the ability space, so as to minimize the distances between the matched points. This is tantamount to matching abilities of people and requirements of tasks.

One of the difficulties is to determine which skills should be considered in the ability space. Consider an individual who is very proficient at playing the piano. However, he has not found a job in the music industry and is presently flipping hamburgers at a local fast food chain. Assume that his cooking skills are sufficient to perform the job, but do not go much beyond that. In the ability space that includes only dimensions for skills that are required by the job, the points representing the employee and the task, respectively, are located closely together, so that it appears a good idea to match the two. This is based on the assumption that a challenging task provides more job satisfaction than an assignment that requires only a small portion of an individual’s ability. In reality, the individual is probably bored to tears, something expressed in an ability space that includes the union of an employee’s existing skills as well as all of the skills required by a task or job.

![Figure 1. The major components of the problem.](image-url)
Another important issue concerns the hiring of new employees who have to be trained before they can be assigned to specific tasks. This is easy to incorporate in dynamic models, but can also be included in this model. Since the hiring process will invariably include some sort of aptitude test, a potential employee’s ability to be trained can be assessed and training costs can be included in the hiring costs. Furthermore, the newly hired employee can be positioned in the ability space where he most likely will be located after the training period.

As far as the abilities are concerned, define $L$ as the set of ability positions $\ell$ with $L = L' \cup L_\phi$, where $L'$ denotes the set of possible final positions in the ability space after training and $L_\phi$ is the outside position, i.e. the pool of employees who are either fired or those who were not hired. Given these definitions, it is now possible to interpret some of the arcs in Figure 1. The arc from $K_e$ to $L'$ represents present employees that are (re-) trained. (Note that the model includes zero training, so that each employee is retrained, where those that do not receive formal training are retrained to their own position, assuming that skills are not lost over time.) The arc leading from $K_e$ to $L_\phi$ symbolizes all present employees who leave the system, i.e. those who are fired or convinced to leave voluntarily. The arc from $K_p$ to $L'$ is used by new potential employees who have been hired, while the arc from $K_p$ to $L_\phi$ includes all potential employees that are not hired.

The third and final set involves all tasks $I$ that are required to be performed. One of the problems involves the level of aggregation that is used. In particular, it has to be decided if, for example, a secretary’s tasks are represented as detailed as ‘typing a memo’, ‘filling out a travel expense claim’ and ‘maintain lists of email addresses’ (i.e. task definitions on the micro level), or if the job is described on the macro level that basically specifies ‘clerical tasks’ and similarly general descriptions. For an in-depth discussion of aggregation of spatial data and problems associated with it, readers are referred to the work Francis et al. (2005). The arc from $L'$ to $I$ indicates the positioning of the ability requirements of the tasks.

Each employee has two sets of restrictions: one concerning the set of tasks he can (presently) perform and another set of abilities he can potentially be trained to.

1. An employee cannot perform tasks that require a higher ability than he has. For technical reasons, we associate ability sets with positions rather than employees, i.e. we define $A_\ell$ as the ability set of position $\ell$.

2. Define the ‘box’ $L_k$ as the set of positions $\ell$ to which employee $k$ can be trained, including his current position and including the outside $L_\phi$, i.e. he is fired. Clearly, $L_\phi \subset L_k \subset L$. We assume that an employee cannot ‘lose’ abilities, consequently, $L_k$ includes only points to the ‘hypernortheast’ of employee $k$’s present position.

The key assignment variables can then be defined

- $x_{k\ell} = 1$, if employee $k \in K$ is trained to position $\ell \in L$ and zero otherwise, and
- $y_{i\ell} \in \mathbb{N}_0$, as the number of times some employee in position $\ell \in L$ performs a task $i \in I$.

The number of variables can, of course, be reduced by defining variables $y_{i\ell}$ only for tasks $i \in A_\ell$, i.e. those tasks $i$ that can be performed by somebody in position $\ell$, and, similarly, define variables $x_{k\ell}$ only for $\ell \in L_k$, i.e. for those employees $k$ that can possibly be trained to position $\ell$. In addition, the following variables will also be needed

- $s_i \in \mathbb{N}_0$ denotes the number of times task $i$ is subcontracted
- $u_\ell \geq 0$ denotes the sum over all employees in position $\ell$ of the time they remain idle during the regular working day; this can be seen as an expression of the total underemployment of existing employees or the total idle time
- $o_\ell \geq 0$ denotes the overtime required in addition to regular working hours of all employees in position $\ell$; again, it is defined as the sum over all employees in position $\ell$. 


Consider now the parameters of the model. First, the cost-related items are

- \( c_{k\ell} \) is the (marginal) cost of employee \( k \)'s salary increase if he is trained to position \( \ell \) plus the training cost, the net present value of the cost of firing (in case the employee is terminated), the hiring cost of a new employee and his salary at position \( \ell \) (note that this can include the seniority of an individual, differences in training costs, etc)
- \( c_{i\ell}^o \) is the cost of each overtime hour of an employee in position \( \ell \),
- \( c_{i\ell}^u \) is the penalty of each hour any employee in position \( \ell \) spends not working on tasks in his regular working day
- \( \hat{c}_i \) denotes the cost of subcontracting task \( i \).

For parameters relating to the working hours of regular employees

- \( w_{\ell} \) denotes the regular working hours of any one employee trained to position \( \ell \)
- \( b_{\ell} \) is the maximum overtime permitted for any one employee trained to position \( \ell \)
- \( t_i \) is the time it takes to perform task \( i \) (assumed equal for all employees). In case there are differences, parameter \( t_{i\ell} \) can be used (defined as the time it takes an employee in position \( \ell \) to perform task \( i \)). Different times will, however, not only change the model but, based on preliminary tests done with such a model, vastly increase the execution times of the model.
- \( f_i \) is the number of times task \( i \) must be performed
- \( d_{i\ell} \) denotes the distance between position \( \ell \) and task \( i \) in the ability space.

Here, the distance between any of the employees in position \( \ell \) and a task \( i \) in the ability space is measured as the average relative unused ability, i.e.

\[
d_{i\ell} = \frac{1}{n} \sum_j [(a_{\ell j} - r_{ij})/a_{\ell j}]
\]

where \( a_{\ell j} \) denotes the ability of an employee in position \( \ell \) with respect to the ability component (dimension) \( j \), \( r_{ij} \) symbolizes the requirement of ability dimension \( j \) for task \( i \) and \( n \) is the dimensionality of the ability space. The magnitude of \( n \) depends on the decision maker’s assessment of which and how many ability components are relevant in the case at hand. Many other distance measures could be chosen, e.g. the very popular class of Minkowski metrics, of which rectilinear, Euclidean and Chebyshev metrics are well-known special cases. This model uses the aforementioned distance measure as it allows a simple and meaningful interpretation.

Finally, define \( b_{kq} \) as the maximal number of times employee \( k \) will perform a task in the set \( I_q \), where \( I_q \) is the set of tasks in category \( q \). The sets collected in each category have similar requirements. The reason for this is that it may not only be desirable to limit the number of times an employee performs identical jobs, but also the number of times an employee performs similar jobs.

The model can then be written as

\[
\begin{align*}
\min z_C &= \text{cost} = \sum_{k \in K} \sum_{\ell \in L} c_{k\ell} x_{k\ell} + \sum_{\ell \in L} c_{i\ell}^o o_{i\ell} + \sum_{\ell \in L_\phi} c_{i\ell}^u u_{i\ell} + \sum_i \hat{c}_i s_i \\
\min z_D &= \text{distance} = \sum_{i \in A_\ell} \sum_{\ell \in L_k \setminus L_\phi} t_{i\ell} d_{i\ell} y_{i\ell} \\
\text{s.t.} \quad &\sum_{\ell \in L_k} x_{k\ell} = 1, \quad \forall k \in K \\
y_{i\ell} &\leq \sum_{k:\ell \in L_k \setminus L_\phi} \left( \min_{i \in I_q} \{ f_i, b_{kq} \} x_{k\ell} \right), \quad \forall \ell, i \in A_\ell
\end{align*}
\]
The first objective minimizes the cost of training, hiring and firing (first term), the cost of overtime (second term), idle time (third term) and the cost of subcontracting (last term). The costs of salaries presently paid is not included, except in the cost of firing, which includes the savings resulting from employees being fired; the objective considers only the marginal costs. Note that the third term in the objective optimizes a measure of efficiency in the allocation of work among employees, since it minimizes the cost of idle time, i.e. the dollar value of negative deviations from the average workload of all employees in position $\ell$. This term can be also considered to optimize equity, as it attempts to distribute the workload evenly among the employees. It only includes idle time as positive deviations are already considered in the overtime cost.

The second objective minimizes the total distance between tasks and positions to which they are assigned. Again, note that while the goal is to match tasks as closely as possible to the employees’ abilities, this model matches tasks to positions. Once the model is solved (resulting in matching tasks to positions), it is then possible to distribute tasks among employees who occupy the same position. In doing so, equity and other features can be taken into consideration.

Another important issue in this context is the type of distance function to be minimized. The model presented here sums the weighted deviations of all positions. Another possible choice would have been to minimize the largest ability–requirement distance with one of the usual minimax functions.

Constraints (1) state that each employee, existing or potential, is trained to some position, possibly his own present position (i.e. no training) or outside (i.e. fired). Constraints (2) state that task $i$ cannot be assigned to position $\ell$ if no employee, existing or potential, is trained to that position. The parameter on the right-hand side is a version of ‘big $M$,’ whose value is as tight as possible: it is the minimum between $f_i$ (no task needs to be performed more than $f_i$ times) and $b_k$ (the maximum number of times employee $k$ will perform a task in the set $I_q$, to which task $i$ belongs). This constraint also acts as an upper bound on the number of times that employee $k$ can perform a task in set $I_q$.

Constraints (3) essentially define overtime hours and negative deviations from the regular workload for position $\ell$. Constraints (4) state that each task must be performed $f_i$ times by an employee or it has to be subcontracted. Finally, constraints (5) provide upper bounds on the overtime of all individuals in position $\ell$.

The problem consists of $|K||L| + |L||I| + |I| + 2|L|$ variables, almost all of which are required to be integers. Note, however, that although variables $s_i$ are integers, they do not need to be declared integers when solving the model since they naturally take integer values when variables $x_{k\ell}$ and $y_{i\ell}$ are integers. Furthermore, the model includes $|K| + 2|L| + (1 + |L|)|I|$ constraints. The constraints that cause difficulties when using an exact solution technique are constraints (2).
Once the problem is solved by whatever means, it is necessary to allocate all tasks that have
been assigned to a specific position to employees who occupy that position. Since it is known
which employees are in each position, overtime can be distributed evenly among employees in
the same position after the above problem has been optimized. Here, this results in

\[ o_k = x_{k\ell} \frac{o_\ell}{\sum_j x_{j\ell}}, \quad \forall k \in K \]

A similar procedure is applied to negative deviations. This will result in some degree of fairness,
as all employees in the same position are treated equally.

As any modeller who faces an optimization problem with multiple objectives, it is necessary to
decide how to deal with a model in which the concept of optimality no longer holds but instead,
there is a set of non-dominated solutions (or a trade-off curve). This model uses the constraint
method (Cohon 1978), consisting of leaving only one objective and transforming the remaining
objectives into constraints. In this case, the cost objective is left as such and the distance objective
becomes the constraint

\[ \sum_{i \in A_i} \sum_{\ell \in L_k \setminus L_\phi} t_i d_{i\ell} y_{i\ell} \leq \bar{D} \quad (6) \]

in which the right-hand side parameter \( \bar{D} \) is varied over the whole range of possible distances.
Each time the problem is solved with a new value of \( \bar{D} \), the left-hand side takes a value \( z_D^* \leq \bar{D} \),
and the objective a value \( z_C^* \). The pair \((z_D^*, z_C^*)\) is a point on the trade-off curve, which is optimal
with respect to one objective given a limit on the other.

Other approaches—such as using a weighting method that minimizes the function
\( z = \alpha z_D + (1 - \alpha)(z_C) \), goal programming, reference point programming and similar modelling
techniques—are possible (for a collection of the major approaches see, for example, Eiselt and
Sandblom (2007) or Collette and Siarry (2003)).

3. Computational results

The model was tested using real data from DICTUC S.A., a company based in Chile that provides
applied research, technology transfer and engineering services, including product testing and
certification. It is also the follow-up service centre in Chile for Underwriters Laboratories (UL)
(further details can be found in Eiselt and Marianov (2008)). The particular case discussed here
was taken from the concrete and construction material-testing lab. The subset considered here
consists of 15 employees, 14 skills and 22 (recurring) tasks, as shown in Tables 1 and 2.
Table 1 displays the tasks, their duration in hours, the number of times they must be performed each day
(frequency), the cost of subcontracting the task and, for each task, the required skill level in each
one of the 14 skills or abilities, ranging from 0–8. The cost of subcontracting, which is unrelated
to the wages of the employees, depends on the complexity of the task and, in general, is more
expensive than having employees performing tasks. Table 2 presents the level achieved by each
employee in each one of the abilities. The last column of the table shows the cost of advancing
one level in training, for each skill. The last two rows indicate the cost of one hour of overtime
for each employee, \( c_o^\ell \), and the cost of firing that employee, respectively. The overtime cost is
typically 1.5 times the cost of regular time, except for extra hours used on Sundays or holidays,
when the cost is twice that of a regular hour. There are three potential new employees, which have
the same ability level, on all skills, as current employees 1, 6 and 15, respectively. These numbers
have been chosen randomly as the skill profiles of potential new employees are not known. In any
case, these three employees represent a wide spectrum of profiles.
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<th>Duration (hr)</th>
<th>Frequency (per day)</th>
<th>Subcontracting cost (US$)</th>
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<tr>
<td>10   General (theoretical) construction practice</td>
<td>2 2 2 2 2 2 2 1 1 1 1 1 1 1 1 1</td>
<td>5768</td>
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<tr>
<td>11   General (practical) construction practice</td>
<td>3 3 3 3 3 3 3 – – – – – – – – – – – – – –</td>
<td>5768</td>
<td></td>
<td></td>
</tr>
<tr>
<td>12   In situ inspection</td>
<td>6 6 6 6 5 5 5 2 2 – – – – – – – – 2 2 2</td>
<td>752</td>
<td></td>
<td></td>
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<tr>
<td>13   Construction laws and regulations</td>
<td>2 2 2 2 1 1 – – – – – – – – – – – – – –</td>
<td>868</td>
<td></td>
<td></td>
</tr>
<tr>
<td>14   Risk prevention</td>
<td>3 3 3 3 3 3 3 2 2 2 2 2 2 2 1 1 2 2 2</td>
<td>1460</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Approximate cost of one extra hour overtime (US$)</td>
<td>15 15 15 15 15 13 12 10 10 10 10 10 10 10 10 10</td>
<td>10</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Daily cost of firing (US$)</td>
<td>60 53 67 59 56 39 40 49 49 44 44 35 35 34 34 37</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
The penalty $c_{\ell}^i$ on the idle time of employees in position $\ell$ is just the same as the regular time ($c_{\ell}^i/1.5$). The limit on regular working hours is $w_{\ell} = 8$, while the limit on overtime hours is $b_{\ell} = 4$.

The model provides the task–employee assignment for a regular working day. It is assumed that the cost of full employee training must be recovered over a year, but expressed in terms of daily wages for modelling effects. The daily hiring cost of new employees is their daily salary plus the daily cost of training required. Hiring costs (daily) are US$80, US$68 and US$41, respectively. The cost of firing includes the cost of advance notice and severance payments and penalties, recovered over a year and expressed in terms of daily wages, minus the daily salary, which is saved when the employee is fired. The number of times that an employee can perform the same task is limited to $b_{kq} = 15$.

Six policies are compared in the experiments. All policies could require existing employees to work overtime, as well as the subcontracting of some tasks.

- **Policy 1—No relocation.** Relocation in skill space is not allowed, i.e. there is no training, hiring or firing. The purpose of this policy is to evaluate the present scenario as a benchmark. This is the most restrictive policy.
- **Policy 2—Training only.** This policy allows training current employees but does not allow firing them or hiring new employees.
- **Policy 3—Hiring and firing only.** This policy does not consider training, but current employees can be fired and new employees can be hired.
- **Policy 4—Training and hiring.** This policy considers that, because of agreements with unions, no current employees can be fired.
- **Policy 5—Training and firing.** This is a conservative policy, for periods of economic instability, when any growth in staff numbers could be risky.
- **Policy 6—Full relocation.** This is policy that allows full relocation in skill space, including training, hiring and firing, i.e. total freedom.

To investigate these six policies, 107 experiments were conducted. For each policy, the entire range of possible distances was covered by using the following set of values for the right-hand side of constraint (6): \{0, 0.01, 0.1, 0.5, 1, 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 50\}. The value 50 was used to find the maximum unrestricted distance in each policy. Additionally, some extra values of this parameter (24, 26, 28, 30, 33) were used in those cases in which the unrestricted distance exceeded 22. For example, Policy 1 required all of these extra right-hand side values, while the remaining policies required between 16 and 18 values, depending on the value of the maximum unrestricted distance.

The linear integer model was solved using CPLEX 9.1, on a Core DUO PC 2.4 GHz, 2 GB RAM memory. For Policy 1, the optimal solution was found in at most 0.63 s, with an average time of 0.1 s. For Policy 2, the solution times were between 0.2 s and 5 s, except for a run that took 4 min 6 s. On average, the time was 1.2 s (not taking into account the single long run). Policy 3 always took less than a second (average 0.23 s); Policy 4 took less than a minute, except for a run that was slightly over 3 hours and another run that took 9 min. In both cases, CPLEX used most of this time to improve the lower bound after finding the optimal solution in less than a few minutes. Eliminating these two runs, the average time was 8.2 s. Policy 5 took 0.2 s to 6 min with an average of 53 s; Policy 6 took 1 s to 17 min, with an average of 1 min 40 s. Disregarding the longest run, the average reduces to 46 s. There does not appear to be any correlation between running time and model parameters.

Figure 2 shows the total cost in relation to the sum of all employee–task distances (total distance) for the six policies. As expected, as policies are more restrictive they become significantly more expensive than the least restrictive Policy 6, even when distances between task requirements and
employees’ abilities were not an issue. For all six policies, reducing employees’ boredom (i.e. task–employee distances) can be achieved but at increased cost. When the first policy is chosen, achieving zero distance is about fourteen times more expensive than the situation in which distance does not matter. If hiring and training, only training, or full relocation are selected, the situation is similar, although absolute costs are reduced by a factor of up to 4 related to the first policy. If only hiring and firing are allowed, the ratio between costs for maximum distance (total boredom) and zero distance (complete fun) is almost 26.

With Policy 1 in place, employee relocation in skill space is not a choice. In other words, a particular employee’s distance to a specific task cannot be changed. If a global distance reduction is desired, the first step is to assign each task to its closest employee until some employees’ working time constraints are binding. Once this assignment is made, the only action that can be taken to further reduce the distance is to break those employee–task assignments with the longest distances, and subcontract the corresponding tasks. As shown in Figures 3 and 4, for the no relocation policy, when zero distance is desired, all tasks are subcontracted and the employees’ workload is zero—an obviously degenerate solution. Naturally, a similar situation occurs when no training is allowed.

This complete workload reduction as a means of reaching zero distance is not needed when training is allowed (as in all the remaining policies), because employees can be relocated in skill space in such a way that they end up closer to the tasks they perform. Figure 3 shows how, as opposed to the case when Policy 1 is chosen, zero distance requires reducing workload only by 30%, approximately, if any of the policies that allow training are chosen. As Figures 3 and 4 clearly demonstrate, the reduction of the average distance in skill space decreases the average workload as more and more tasks (especially those whose skill requirements are quite different from those available) are subcontracted.

In addition to workload, it is interesting to compare the idle time cost for the policies. Figure 5 shows how the hiring and firing policy makes more efficient use of employees’ time, followed by the firing training and the full relocation policies, which are very close to each other. The efficiency of the hiring–firing policy is achieved by ‘packing’ the tasks so as to keep every employee busy, and firing those that are not. In addition, it uses subcontracting for those tasks that cannot be

Figure 2. Cost in relation to total distance for the six policies. Available in colour online.
performed by employees. The training only and the hiring–training policies are close to each other, but costlier in terms of idle time (they trade-off idle time for subcontracting). The most expensive policy in terms of idle time is, by far, the status quo policy, i.e. the no relocation policy.

The overtime costs are shown in Figure 6, for three of the policies, since for the remaining policies, these costs do not follow clear patterns.

Consider now the ‘no relocation’ policy. When distance is not an issue, the fact that subcontracting is very expensive implies that all tasks are performed by current employees. In this situation, many employees will be performing tasks whose skill requirements are probably far from their
ability levels. Also, since there are some tasks that can be performed only by certain employees (probably the most skilled ones), these employees must work overtime, while other employees (those less trained) remain idle part of the time. Thus, overtime grows with distance.

When distance becomes important, i.e. it must decrease, overtime is not a choice. As skilled employees must carry on performing simple tasks during overtime, this strategy increases the total distance. When tasks are subcontracted, employees do not work overtime any longer. Furthermore, as distance decreases, employees’ idle time increases and overtime becomes zero.

This is not the case with the policies that allow training, because employees can be trained to be ‘close’ to the tasks they perform. The more demanding distance requirements are fulfilled as far as possible by overtime work of some of the trained employees, which is the cheapest strategy.
The relation between the costs of relocating (training, hiring, firing) and the average distances between employees and tasks is depicted in Figure 7. The ‘no relocation’ policy has zero relocation costs and is not considered in this figure. Note that negative costs correspond to savings obtained by firing employees; these savings are considerable when all employees are fired under the hiring and firing policy. Of course, these savings are balanced by the subcontracting costs. When policies allowing training are chosen, there is an increase in training costs as the distances are required to decrease—the cheapest action for achieving these short distances is to train employees.

No convexities in the curves are due to granularity of costs. An example of granularity is shown in Table 3, in which fired and hired employees are listed. Note that, as distance is required to decrease, more expensive employees are fired and least expensive candidates hired. This is due to the fact that large savings are obtained by firing expensive employees and replacing them with less expensive candidates that go through cheap training.

### Table 3. Firing and hiring in Policy 6.

<table>
<thead>
<tr>
<th>Distance</th>
<th>Fired</th>
<th>Hired</th>
<th>Difference</th>
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</thead>
<tbody>
<tr>
<td>≤0.5</td>
<td>1, 2, 3, 9</td>
<td>18</td>
<td>−3</td>
</tr>
<tr>
<td>4</td>
<td>1, 2, 3, 9</td>
<td>17, 18</td>
<td>−2</td>
</tr>
<tr>
<td>6</td>
<td>1, 3, 4</td>
<td>17, 18</td>
<td>−1</td>
</tr>
<tr>
<td>10</td>
<td>1, 3, 9</td>
<td>17, 18</td>
<td>−1</td>
</tr>
<tr>
<td>12</td>
<td>1, 3, 9</td>
<td>17, 18</td>
<td>0</td>
</tr>
<tr>
<td>14</td>
<td>2</td>
<td>17, 18</td>
<td>1</td>
</tr>
<tr>
<td>18</td>
<td>2, 9</td>
<td>17, 18</td>
<td>0</td>
</tr>
<tr>
<td>20</td>
<td>9</td>
<td>17, 18</td>
<td>1</td>
</tr>
<tr>
<td>25</td>
<td>8, 9</td>
<td>17, 18</td>
<td>0</td>
</tr>
</tbody>
</table>

4. Conclusions, summary and outlook

This article has presented a model that included components that are not only relevant to a firm, *i.e.* the planner, but also to those involved in the execution of the plan, *i.e.* the employees. Their interests...
regarding challenges presented by the tasks are directly taken into consideration and their costs are determined. A real-life example was solved and numerous scenarios were evaluated. This example demonstrated that realistic problems of this nature are solvable within a reasonable amount of time.

Beyond the obvious results—such as the expected outcome that costs will increase as closer matchings between employees and tasks are forced—the article demonstrates an inverse relationship between the employee–task distance and idle time. While this relationship is easy to explain, it does open up the possibility for some compromise. In particular, it indicates that while close proximity of an employee’s abilities to the requirements of a task is desirable, very long idle times are not. Long idle times convey a sense that an individual’s services are irrelevant, indicate the possibility of being laid off and imply an employee has little chance of moving up in the corporate hierarchy. This suggests that a balance between idle time and employee–task distance is desirable—a feature not explicitly included in the model. It is also apparent that the status quo option, i.e. a no relocation policy, is very expensive in comparison with all other policies. Furthermore, a status quo policy and a policy that allows hiring and firing result in large numbers of jobs being subcontracted.

These observations lead directly to potential future research. The value of training could be investigated. This is, of course, a well-known area of study, see, e.g. Law (1995). However, the spatial modelling of abilities can bring a new aspect to usual research in the field. Another line of research evolves when allowing the possibility that employees can lose some of their abilities over time, particularly if they do not perform tasks that require these abilities. Such instances are common when new technologies are introduced, e.g. new biochemical testing procedures, updated software for office management, etc. The introduction of such loss of ability (or, similarly, the obsolescence of existing knowledge) is an important feature that may be included in a different model. Other possible extensions could put the different interests of employees and the firm in a game-theoretic context that pits labour against management and attempts to determine pockets of compromise. Finally, it would be very interesting to study a dynamic model to investigate whether or not a firm would, as the cost structure may suggest, try to hire new (and less expensive) employees while firing older, more expensive employees.

Another interesting aspect is the usability of this model from an employee’s perspective: it may provide him (or a ‘headhunter’ or similar management human resource consulting firm) to design an optimal personalized training plan so as to maximize his own marketability.

Another thought concerns the computability of large-scale problems. While the example in this article has shown that the computation times are not prohibitive, they may quickly become unmanageable as the problem size increases. Heuristics, e.g. those that use allocation rules such as fixed assignments for some employees that are known to perform well in certain positions, can be used to reduce the size of the problem and make it solvable, albeit at the price of a suboptimal solution.

Acknowledgements

This article was in part supported by a grant from the Natural Sciences and Engineering Research Council of Canada under grant #9160. This support is gratefully acknowledged. The second author thanks DICTUC S.A. for permission to use data in computational experiments, as well as grant FONDECYT 1070741.

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