Employee positioning and workload allocation

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Abstract

Assigning tasks to employees is a difficult task. Errors committed in such assignments can have far-reaching consequences, such as reduced efficiency due to absenteeism, lack of job satisfaction, formal grievances, and generally deteriorating labor relations. This paper approaches the problem from a spatial point of view. First, the employees and the relevant tasks are mapped in a skill space. After feasible task assignments are determined, tasks are assigned to employees so as to minimize employee—task distances in order to avoid boredom, and minimize disequity between the individual employees’ workloads, and minimize costs. Computational results are provided for an engineering department of the Pontificia Universidad Católica de Chile in Santiago, Chile.

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1. Introduction

Each year, the economy suffers from losses due to unscheduled absences by employees. A recent figure pegs the annual cost of both paid and unpaid absenteeism per employee at between $400 and $700 for companies with 100–249 employees [1]. (The corresponding figure for smaller companies is substantially lower.) Given tens of millions of employees in the economies of developed countries, this adds up to hundreds of millions of dollars due to absenteeism alone. Absenteeism is based on workers having utility functions, which have a higher utility for being absent than the (potential) penalty that is associated with it. A high utility for being absent indicates that there is a major discrepancy between an employee’s job satisfaction at work and alternative activities.

There is a wealth of findings in the field of job satisfaction and its correlation (or the lack thereof) with absenteeism and labor turnover. Most research is based on the groundbreaking work by Herzberg [2] and his two factor theory that distinguishes between motivators (i.e., factors whose presence matters, but whose absence does not) and hygiene factors (i.e., factors whose presence does not matter, but whose absence is detrimental to job satisfaction). The early work by Talacchi [3], Vroom [4], and Locke [5] see an inverse correlation between job satisfaction and absenteeism. Most later authors confirm this, e.g., Scott and Taylor [6], Petterson et al. [7] and Roessler et al. [8]. However, researchers such as Bassett [9] point out that many factors determine absenteeism, e.g., job design, supervision, compensation, degree of repetition required in the work (which may be viewed positively or negatively by different employees). There does, however, appear to be consensus that there is a moderate correlation between job satisfaction and absenteeism.

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and turnover. This aspect is incorporated in our paper. The aforementioned repetitiveness is also included as a set of constraints.

However, absenteeism is but one reason for the lack of efficiency in businesses. Another reason whose effects are much harder to quantify is the misassignment of employees to jobs. In other words, given the employees available within an organization, tasks may have been (and frequently are) assigned to individuals who are either not qualified to do the job, or have personal reasons to perform the job in a non-efficient manner. Jealousy, turf protection, and plain disinterest in the task are only some of the reasons for such misassignments.

Tools to overcome some of the problems are manifold. Motivational strategies, efficiency bonuses, and penalties for non-compliance are among the devices available to the personnel manager to avoid inefficiencies.

The thrust of this paper is to work with the tasks and employees available to the organization as much as possible and allocate work so as to increase an employees job satisfaction, thus narrowing the gap between the employee’s utility of going to work and the alternatives, and hence, ceteris paribus, reducing absenteeism and similar inefficient behavior.

Staff assignment, workload allocation, and rostering (personnel scheduling) problems are by no means new. Roberts and Escudero [10] assign employees with different skills to tasks so as to minimize employees’ idle time. A sample of applications includes nurse schedules [11], maintenance workers [12], Navy personnel [13], and employees of a power station [14]. Each of these studies includes different abilities and skill requirements of jobs. For general surveys on nurse rostering and staff scheduling, readers are referred to Cheang et al. [15] and Ernst et al. [16], respectively. Solution methods of choice are typically column generation, Lagrangean relaxation, network models (such as [17]), general integer programming, and heuristics.

What distinguishes this study from the aforementioned work is that we do not include scheduling in our work, but instead investigate multidimensional skills (along with their spatial representation) and use multiple objectives. This is a step away from the operational task of answering the “who does what and when?” questions and a move to the more tactical task of personnel retention through long-term objectives such as perceived equity of workload distribution and the assignment of tasks according the employees’ abilities.

The remainder of this paper is organized as follows. Section 2 develops the model and analyzes its input parameters. Section 3 presents the formal mathematical model, while Section 4 applies the formulation to a real-world problem. Finally, Section 5 summarizes the paper and outlines some of the potential extensions of our approach.

2. The model

In this section we develop the model that is formally presented in the next section. The basic focus of this paper is job satisfaction (or the lack thereof). Factors that influence job satisfaction can be subdivided into individual factors and collective factors. Individual factors are those that exist in isolation, i.e., factors that concern only the employee himself, whereas collective factors are those, in which an employee compares his own situation with that of other employees (typically in the same organization, department, or group).

As far as individual factors are concerned, one concerns boredom with the job, and another an employee’s workload. Collective factors include the disparity of workload. Below, we will deal with each of these two factors separately.

First consider boredom. There are two sources of boredom: the variance or repetition of tasks, and the challenge that a job provides. In order to control the repetition of tasks, we can formulate a constraint that provides an upper bound on the number of times that the same individual performs the same or a similar task. This will increase the variance of the jobs assigned to an employee.

Consider now the challenge that a job provides to an individual. A challenging activity will keep an employee interested in his job, and, as an added benefit, possibly provide opportunities to move up in the hierarchy. Measuring challenge is, however, a difficult problem.Crudely speaking, a challenge can be expressed as the discrepancy between an employee’s abilities and the job requirements.

In order to explain this concept, consider the following example. There are three employees A, B, and C, and twelve tasks numbered 1, 2, . . . , 12. Two skills have been identified as relevant, and employees have acquired them at different levels, through formal training and experience. Level 1 is the lowest, follows level 2, and so on. Employees have been classified according to the level they have reached in each skill. Also, in order to be performed correctly, each task requires possession of each skill at a certain level. The skill levels of the employees as well as the skill levels required by the tasks are shown in Fig. 1, where the abscissa values correspond to levels of skill 1, and the ordinate values are levels of skill 2.
The first consideration involves feasibility. Clearly, due to their lack of skills, the existing employees cannot perform tasks 7, 8, 9, and 12. The remaining tasks can be performed by at least one employee. For simplicity, we have indicated in each of the six rectangles which employees can perform the tasks in that rectangle (including its northeastern boundaries). For instance, task 11 can only be performed by employee A, and task 6 can only be performed by employee C. Similarly, employees A and B can both perform tasks 2 and 10, while C cannot. Given the mandatory task assignments 11 to A and 6 to C, there are many degrees of freedom. For instance, it would be feasible to assign tasks 1, 2, 3, 4, 10, and 11 to employee A, while asking C to perform tasks 4, 5, and 6. This leaves B idle, an obviously undesirable, albeit feasible, assignment. Similarly, assigning task 1 to employee A does not appear to be a good idea, as A’s skills are much higher than those required by task 1, thus resulting in boredom, which is precisely what we try to avoid. A better choice appears to be to assign task 1 to B, whose skills are also significantly higher than those required, but less so than those of A or C.

This discussion clearly indicates the need for a quantitative measurement of skill levels and a metric that measures differences in skill levels. One immediate idea is to define a (suitably modified) Voronoi diagram, which uses the employee points in skill space as seeds and allocates tasks to the nearest employee, according to some pre-chosen metric. Here, it is possible (and often desirable) to assign distinct weights to the individual skills. Such a procedure will lead to assigning tasks to their closest employees, as is the case in $p$-median problems. Typically, such assignments will exhibit large differences in individual workloads, thus requiring additional considerations.

An important question is which abilities are included in the model. A task-centered approach will include all abilities that are required by the task, while an employee-centered approach will include all abilities possessed by an employee. In order to explain the difference between the two approaches, consider the job of a professional photographer who happens to play the piano very well. Assume that his photographic abilities are adequate. The task-centered approach will map the daily work and the photographer’s abilities in the single dimension of photographic ability, in which the distance between the job requirement and the photographer’s abilities may be fairly small, thus indicating satisfaction. On the other hand, the employee-centered ability diagram will include an additional dimension for abilities on the piano, with the pertinent requirement of the job being naturally zero, while the employee’s ability on that dimension may be
very high. This will result in a substantial distance between the task and the employee, symbolizing dissatisfaction. Here, we argue that in case of major discrepancies between an employee’s abilities and a task’s requirements, the individual would most likely not have chosen the job in the first place, or, at least, only temporarily. As a result we have chosen to apply the task-centered approach.

As far as the individual workload is concerned, there will have to be an obvious penalty for overtime, which can be interpreted as overtime cost. However, if employees are paid for full days, there is also a cost associated with employees working less than full time, as this again contributes to boredom and because employees are being paid for not doing any work during their idle time. These penalties can and usually will be different from each other.

Compare now an employee’s individual workload to that of other employees. If the goal is to provide workloads that are “as equal as possible”, then an “equity objective” is called for. Objectives of this type have been discussed by many authors, see, e.g., Savas [18] and Truelove [19], and, in the locational context of equity objectives, Marsh and Schilling [20] and Eiselt and Laporte [21]. Among the more popular measures figure the variance and the Gini index and their derivatives. In a mathematical model, equity objectives can either be formulated as a (probably non-linear) objective function, or they are included in the model in the form of constraints. These constraints simply provide lower and upper bounds on the number of hours an employee will work. This is probably the easiest way to handle them. Other possibilities are to minimize the maximum deviation from the average workload, where either both positive and negative, or only positive deviations from the average are counted.

So far, the problem under consideration is static in the sense that skill levels of employees and skill requirements of tasks are fixed, and there are no additional employees to be hired or tasks subcontracted (corresponding to optimally locating a new employee in the skill space), or optimal (re-)training of existing employees (i.e., relocating existing employee locations in skill space).

In practice, if all tasks need to be performed, either additional help is hired through some form of contracting out, or some of the existing employees are trained in order to be able to perform the tasks that they are not possible to perform at present.

When retraining, we have again the choice of a variety of techniques. Two of them stand out prominently: on the one hand, there is the employee-centered learning, a technique that is based on individual instruction in the sense that it takes into account the prior knowledge of an employee. Such instruction can be performed by any medium, either human instructor, or books, or web-based instruction. This type of instruction carries costs that are more or less proportional to the distance between an employee’s abilities and a task’s requirements.

On the other hand, there is task-centered instruction. This type of instruction is typically a course, in which the instructor starts the course at an agreed-upon level and brings all participants to the level required by the task (or group of tasks). The costs of this approach in the standard approach of group instruction would most likely be proportional to the distance between the task and a point in the ability space that represents the minimum of each of the individual abilities of the employees participating in the course. In our work, we choose the former approach.

When training is a possible choice, there are some additional issues. One such issue concerns the probability with which the employee acquires the desired level of skill. In other words, employees can fail the retraining course. This probability can be explicitly included in the model as a “mark-up” in the training cost, which can depend on the worker or the ability. Another issue is related to the decision about which employees should be engaged in training programs. Should the less skilled workers be trained, because the worker-task assignment choices increase and these workers become busier, or should the more skilled workers be trained, because this can be done at the least cost? We leave this decision to the model.

### 3. Mathematical formulation of the model

This section develops a formal mathematical model based on the discussion in the previous section. In our model, we consider tasks $1, \ldots, i, \ldots, m$, employees $1, \ldots, k, \ldots, p$, and abilities/skills $1, \ldots, j, \ldots, n$. We will then require the following parameters:

- $r_{ij}$ requirement of task $i$ regarding skill $j$. They are collected in the matrix $R = (r_{ij})$.
- $a_{kj}$ ability of employee $k$ regarding skill $j$. They are collected in the matrix $A = (a_{kj})$. 
Given the matrices $R$ and $A$, we can determine “distances” between a task and a worker. In general, any metric will do, and we will discuss specifics below. For now it is sufficient that we define $d_{ik}$ as the distance between task $i$ and employee $k$ in the ability space. The parameters $t_i$ denote the duration of task $i$; they are collected in the vector $t = (t_i)$. The parameters $f_i$ denote the number of times task $i$ must be performed. Furthermore, we can define the set of feasible tasks (the “ability set”) of employee $k$ as $A_k = \{i : a_k \geq r_i\}$. This set is computed prior to any optimization. Finally, we define integer variables $x_{ik}$ that represent the number of times task $i \in A_k$ is assigned to employee $k$.

First consider the individual distribution of the workload. Defining $W_k$ as the actual workload of employee $k$, we can write

$$W_k = \sum_{i} t_i x_{ik} \quad \forall k.$$  

With $\ell_k$ denoting an upper bound on the regular work hours of employee $k$, and $o_k$ as the actual overtime hours worked by employee $k$, we define the overtime as

$$W_k \leq \ell_k + o_k \quad \forall k.$$  

If the actual workload $W_k$ is below the bound $\ell_k$, then $o_k$ can assume the value of zero, which it will, since its cost contribution in the objective function is positive. However, since the overtime that an employee can work is limited, we need to define $b_k$ as an upper bound on employee $k$‘s overtime, so that we can formulate

$$o_k \leq b_k \quad \forall k.$$  

Furthermore, the contribution of employee $k$’s wage to the cost-minimizing objective function is $c_k o_k$, where $c_k$ denotes the hourly overtime pay of employee $k$.

Consider now the disequity concerning the distribution of the workload. Defining $W = (1/p) \sum_k W_k$ as the average workload of any employee in the unit, $W_k - W$ denotes the deviation of employee $k$’s workload from the average. It is possible to define different “costs” or penalties $c_k^+$ and $c_k^-$ associated with positive and negative values of $W_k - W$, where positive deviations symbolize the fact that the employee $k$ is busier than the average employee, while negative deviations indicate relative idleness or low activity of an employee. Both can be seen as indicators of unfairness in the assignment of tasks to employees. We can minimize weighted positive and negative deviations together in the equity (or “unfairness”) objective

$$z_E = \sum_k c_k^+ w_k^+ + \sum_k c_k^- w_k^-,$$

where, as under- and overachievements in goal programming, we define

$$W_k + w_k^- - w_k^+ = W \quad \forall k.$$  

The second main issue to be dealt with is boredom. The variance of the work can be expressed as the amount of time that an employee is performing similar jobs. In order to do so, define sets $S_q$, so that all jobs in one of these sets have similar characteristics. We can then require that

$$\sum_{i \in S_q} t_i x_{ik} \leq T_{kq} \quad \forall k, q,$$

i.e., the amount of time employee $k$ spends on jobs in class $S_q$ does not exceed a preset limit $t_{kq}$. Equivalently, we can limit the number of times the employee performs tasks in the same class, through the constraint

$$\sum_{i \in S_q} x_{ik} \leq b_{kq} \quad \forall k, q,$$

where $b_{kq}$ is the maximum number of times employee $k$ can be assigned to tasks in the same class $S_q$. Note that although this limit is introduced to avoid boredom, it cannot be chosen to be too small, i.e. few repetitions, because repetitions help the employees to keep the skills needed to perform the tasks in the class $S_q$. If the limit is small, the skill might be lost by the employee; if the limit is too high, the skill is maintained, but the employee becomes bored. Finally, consider
the challenge that the tasks represent to the employee. The boredom (or disutility) of a worker \( k \) who is assigned to a task \( i \) at least once consists of two parts:

\textbf{Part 1:} The disutility of assigning task \( i \) to employee \( k \) is some (distance) function that relates the worker’s abilities \( a_k \) and the requirement of the task \( r_i \). This discrepancy is measured as some distance function \( d_{ik} = f(r_i, a_k) \).

\textbf{Part 2:} The total boredom of a worker is some aggregation of the individual ability—requirement differences. We will aggregate the individual disutilities as

\[ D_k = g(\bullet) = \sum_{i} t_i d_{ik} x_{ik}. \]

As far as Part 1 is concerned, we have a number of options. One possibility is to employ one of the usual Minkowski \( \ell_p \) metrics. In our model, we then obtain

\[ \left( \sum_{j=1}^{n} (a_{kj} - r_{ij})^p \right)^{1/p}. \]

For \( p = 1 \), we obtain the Manhattan metric, which, as Eiselt and Eiselt [22] have pointed out, indicates that the differences between the available and required skill levels accumulate, whereas for \( p \to \infty \), the Chebyshev distance results, signifying that the difference between the available and the required skill levels is considered just as large as the largest individual difference. Finally, for \( p = 2 \), we obtain Euclidean distances, which represent a compromise between the aforementioned two norms, in that they tend to amplify (by squaring) longer distances, thus giving them added prominence. Another possibility is to use the average relative unused ability

\[ d_{ik} = \left( \frac{1}{n} \sum_{j} \frac{(a_{kj} - r_{ij})}{a_{kj}} \right). \]

Some additional definitions are required. Let \( s_i \) be an integer variable that represents the number of times task \( i \) must be contracted out, if none of the regular employees performs this task in regular or in overtime. Every time task \( i \) is contracted out, costs \( \hat{c}_i \) are incurred. Then the problem can be formulated as follows:

\[ \text{Min} \quad z_C = \sum_{k=1}^{p} c_k o_k + \sum_{i=1}^{m} s_i \]

\[ \text{Min} \quad z_D = \sum_{i \in A_k} \sum_{k=1}^{p} t_i d_{ik} x_{ik} \]

\[ \text{Min} \quad z_E = \sum_{k} c_k^+ w_k^+ + \sum_{k} c_k^- w_k^- \]

s.t.

\[ \sum_{i} t_i x_{ik} \leq \ell_k + o_k \quad \forall k, \quad (1) \]

\[ \sum_{k} x_{ik} + s_i = f_i \quad \forall i, \quad (2) \]

\[ \sum_{i \in S_k} x_{ik} \leq b_{kq} \quad \forall k, q, \quad (3) \]

\[ o_k \leq b_k \quad \forall k, \quad (4) \]

\[ W_k + w_k^+ - w_k^- = W \quad \forall k, \quad (5) \]

\[ x_{ik}, s_i \in \mathbb{Z} \quad \forall i, k, \quad (6) \]

\[ w_k^+ , w_k^- \geq 0 \quad \forall k. \quad (7) \]

The first objective \( z_C \) is the cost of paying overtime hours \( (o_k) \) and contracting out some of the tasks. The second objective \( z_D \) is the total distance between tasks and workers, where distance is measured as desired by the decision maker. The third objective \( z_E \) measures the equity, i.e., the total deviation from the average workload. Constraints (1) limit the regular working time of each worker \( k \) to \( \ell_k \) hours, and count the needed overtime of the same worker. Constraints (2) ensure that each job \( i \) is performed exactly the number of times \( f_i \) that it needs to be performed, either by regular employees, or contracted out. Constraints (3) limit the amount of time an employee can perform tasks from the same class, and constraints (4) limit the overtime of a regular employee. Finally, constraints (5) measure the deviations from the average workload for each employee.
Given this multiple-objective integer linear optimization problem, we combine the objectives by using the weighting method (see [23]) using parameters $x$, $\beta$ and $\gamma$ that are set by the decision maker, and varied throughout the experiments. That way, we obtain the objective

$$
\text{Min } z = \sum_{k=1}^{p} c_k a_k + \sum_{k=1}^{p} \hat{c}_k s_k + \beta \left[ \sum_{i \in A_k} \sum_{k=1}^{p} t_i d_{i,k} x_{i,k} \right] + \gamma \left[ \sum_{k} c^+_k w^+_k + \sum_{k} c^-_k w^-_k \right].
$$

4. A real-world problem

Our test data are based on a real-world problem. The data were taken from DICTUC S.A., a company owned by the Pontificia Universidad Católica de Chile, that provides applied research, technology transfer and engineering services, including product testing and certification, as for example, follow-up services for Underwriters Laboratories, Inc. The company was founded in 1938 as an office at the School of Engineering, and it has presently some 300 employees, one-third of whom are professors of the University. The raison d’être of the office in the early days was to finance expensive lab equipment of the University (which was achieved by a profit-sharing plan, in which the University acts as the professors’ agent), and, later, the technology transfer from classroom to lab.

The problem we will discuss here was taken from the concrete and construction material-testing lab. At present, this lab employs about 50–60 full time people. We look at a subset of 15 employees, 14 skills, and 22 (recurring) tasks, as shown in Tables 1 and 2. Table 1 shows the tasks, their duration in hours, the number of times it must be performed each day (frequency), the cost of subcontracting the task and, for each task, the required ability level, ranging from none to level 8, in each one of the 14 skills. The cost of subcontracting is unrelated to the wages of the employees, it depends on the complexity of the task, and in general, it is more expensive than having the employees performing the tasks. Table 2 shows the ability level achieved by each employee in each one of the skills. The cost of the hour of overtime for each employee is shown in the last column of Table 2. It is typically 1.5 times the cost of regular time, unless the extra hours are used on Sunday or holidays, in which case the cost is twice that of a regular hour.
The distance between each employee and the tasks in his ability set are computed as the average relative unused ability, i.e., $d_{ik} = (1/n) \sum_j [(a_{kj} - r_{ij})/a_{kj}]$. Individual costs on deviations from the average workload ($c_k^+$ and $c_k^-$) were set to 1. The limit on regular working hours is $\ell_k = 8$, while the limit on overtime hours is $b_k = 6$.

Note that we are not considering either hiring new employees or firing existing ones as valid options. This is because we assume that the current staff is optimal in the long term, and we are solving the short term problem, i.e. possibly a few months. However, hiring and firing is being considered in further research.

We run the experiments using CPLEX 7.5, on a Pentium IV PC 2.5 GHz, 500 MB RAM of memory. The values of several parameters were swept to reflect the effects of different policies. In different runs, we limited the number of

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times that an employee can perform the same task to $b_{kq} = 5, 10$ and 20. For values of $b_{kq}$ between 10 and 20, the results do not change significantly.

The weight on the distance objective was set to $\beta = 0.1, 0.5$, and 0.9 and one extreme case was run with a weight of 0.99, while the weight on the cost objective was set to $x = (1 - \beta)$. The weight on the equity objective $\gamma$ took values from 0.01 to 50, and one run was done with a value of 0.

In general, the solution of the model was found in times ranging from 0.1 s to 3 or 4 min. In one case the run was interrupted after reaching a solution within 0.18% of the bound, which happened after running for 5 min.

Fig. 2 shows how both the average workload and the total positive deviation from average workload decrease, as we change the value of the weight on the equity objective, $\gamma$, and keep fixed and equal to 5 the limit on the number of times an employee is assigned the same task. As the weight on the unfairness (or equity) objective increases, the individual workload of employees approaches the average value, and the deviations from this average decrease. Note that, as we increase the value of $\gamma$, thus increasing workload equity, not only the deviations from the average overload decrease as expected, but also does so the average workload, due to two causes. The first cause is the limit on the number of times an employee is assigned the same task, $b_{kq}$, and the second one is the natural upper limit in the working hours of less skilled employees. In fact, these employees can perform only simple tasks due to their lack of the abilities needed to perform the more complicated jobs. Furthermore, as they can perform each task only a limited number of times, this imposes an upper bound in the hours they work that is dictated by the total time needed for the simpler tasks. In the case at hand, this upper bound is below the normal eight working hours, closer to 6 h. As equity becomes more important, i.e., the weight $\gamma$ on the equity objective increases, the number of hours worked by these less skilled employees cannot increase, and the working time of the more skilled employees must decrease, in order to increase equity.

Fig. 3 shows how the subcontracting and overtime costs change, as we change the value of the weight on the equity objective $\gamma$, while keeping fixed and equal to 5 the limit on the number of times an employee can perform the same task. As the weight on the equity objective increases, individual employees’ workloads approach the average value, this average value decreases (as shown in Fig. 2) and the total overtime (and its cost) decreases and more tasks must be subcontracted, increasing the corresponding subcontracting cost.

For the same runs, i.e., varying $\gamma$, Fig. 4 shows the changes in average distance as measured by the distance objective. The decrease in distance is due to the extra assignment freedom obtained by subcontracting more tasks, at an increased subcontracting cost.

When a higher limit $b_{kq}$ is imposed on the number of times a task can be assigned to the same employee, the same shape of curves is obtained. When this limit is 10, ignoring a slow run (22 min) with a very large weight (30) on the equity objective, the average time it took to find the optimal solution was 36 s. The average workload changed less than before; this time the minimum value is 7.03 h as opposed to 5.9 in the previous case. The maximum value is 8.23 h (including overtime). The need for subcontracting in order to increase the equity is also reduced.
Finally, Table 3 and Fig. 5 show a set of results for different values of the weights on the distance and cost objectives, when the weight on the unfairness objective remains unchanged and equal to 1. The limit $b_{kq}$ is 10.

For $b_{kq} = 10$ and 20, with a small weight $\beta$ on the distance, one of the employees does not work at all, while another employee works for 14 h, which is obviously undesirable and unsustainable.

5. Summary and conclusions

In this paper we have proposed a model for the assignment of tasks to individual employees, when several goals are to be considered, and when there are constraints posed by employees’ capabilities. We define a skill space, in which each dimension represents a skill or ability. Each employee can be mapped into this space, his position representing the level acquired in each skill. Similarly, tasks can also be mapped into the skill space, and their position will represent the required level in each skill. After feasible task assignments are determined, tasks are assigned to employees. Employees can work overtime and tasks can be subcontracted, both at known costs. The model in its present form applies to the case in which all tasks must be performed, some of them could be repetitive, and the objectives are to minimize disequity between the individual employees’ workloads, minimize employee—task distances in order to avoid boredom, and minimize overtime and subcontracting cost. Computational results are obtained for a set of employees working in a test lab at DICTUC S.A., a university-owned company in Santiago, Chile, where 15 employees perform 22 tasks that
require 14 skills at different levels of ability. Different solutions are proposed to the decision maker, who can decide the assignments according to the relative importance of each objective.

A possible extension of the model includes the retraining of employees, i.e., their relocation in the skill space. Training aspects were included, for instance, in the aforementioned Liang and Buclatin [13] study. Different types of retraining—task-centered or employee-centered, standardized or specialized—are presently under investigation.

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