Torsional balance of plan-asymmetric structures with frictional dampers: analytical results

Juan C. de la Llera*,†, José L. Almazán and Ignacio J. Vial

Department of Structural Engineering, Pontificia Universidad Católica de Chile, Casilla 306, Correo 22, Santiago, Chile

SUMMARY

This investigation deals with the torsional balance of the earthquake response and design of elastic asymmetric structures with frictional dampers. Plan asymmetry leads to an uneven lateral deformation demand among structural members and to unbalanced designs with larger capacities in some resisting planes. Frictional dampers are capable of controlling lateral-torsional coupling by placing the so-called empirical center of balance (ECB) of the structure at equal distance from all edges of the building. This rule is developed for single-story systems with linear and inelastic behavior. However, recently obtained theoretical and experimental results demonstrate that this rule carries over to multistory structures. Results show that the peak displacement demand at the building edges and that of resisting planes equidistant from the geometric center may be similar if the damper is optimally placed. It is also shown that torsional amplification of the edge displacements of arbitrary asymmetric structures relative to the displacement of the symmetric counterparts are approximately bound by a factor of 2. Furthermore, frictional dampers are equally effective in controlling lateral-torsional coupling of torsionally flexible as well as stiff structures. Copyright © 2005 John Wiley & Sons, Ltd.

KEY WORDS: torsional balance; plan asymmetry; frictional dampers; earthquake response; empirical center of balance; response reduction factors; torsional index; static eccentricity

INTRODUCTION

For a building subjected to a ground motion, plan asymmetry leads to an uneven lateral deformation demand among structural members. The response of plan asymmetric structures during previous earthquakes has shown that the elastic and inelastic deformation demand may
tend to concentrate in a few resisting planes [1]. As a result, design codes incorporate procedures to account for such irregular plan-wise displacement distribution, leading to different stiffnesses and capacities of resisting planes.

But instead of accounting for it, is it possible to balance the torsional response of an asymmetric structure with passive energy dissipation (ED) devices?, what options do we have to control lateral-torsional coupling? The answer to the first question is affirmative and the alternatives to do so are multiple. In this research, frictional dampers are investigated and used to control the torsional response of a structure. This investigation was motivated by the fact that frictional dampers enable us to dissociate stiffness and strength. Previous research [2] showed that by controlling the strength of resisting planes, the inelastic properties of the building represented by the ultimate story shear and torque surface may be modified in order to reduce the inelastic rotations of the building plan. Once lateral-torsional coupling is controlled in the structure, the problem transforms into that of a nominally symmetric structure, implying simpler design procedures, more efficient use of structural members, and more reliable structures.

Consequently, the earthquake response of asymmetric structures with supplemental frictional dampers (FD) as a secondary structural system is investigated. Three aspects are of primary interest: (i) the optimal supplemental dissipation capacity used in the design of the structure, (ii) the optimal location of the dampers in plan, and (iii) the response reduction factors (performance) attained by the use of these optimal supplemental capacities and locations in plan. The concept of torsional balance is introduced by considering the response of simple single-story structural systems and, in a sequel article, the experimental behavior of multistory structures, which validates all theoretical aspects stated in this article.

Several examples of frictional devices to control the earthquake response have been proposed in the past [3–7]. Since the principle of dry friction is used in all of them and due to its simplicity to implement experimentally, the slotted bolted connection (SBC) with Teflon® and polished steel interfaces will be considered in this research. Results are by no means restricted to a specific device as long as the stick-and-slip phenomenon is minimized.

Several parametric studies of one-story asymmetric systems considering energy dissipation devices have been completed recently [8–10]. First, the elastic behavior of asymmetric structures with viscous supplemental dampers was considered. Different values of the damping ratio $\xi_{SD}$, radius of gyration of supplemental damping, $\rho_{SD}$, and eccentricity parameter $\epsilon_{SD}$ for the dampers were used for structures subjected to the NS component of the Sylmar record (Northridge, 1994) [8]. It was shown that the use of viscous supplemental damping may reduce the deformation demand by up to three times if a correct selection of the system parameters is made. More recently, the inelastic behavior of the primary structure using a bilinear model was also considered [9]. It was shown that the supplemental dampers reduce the deformation demand and the ductility demand of the primary structural members; the results were consistent with those obtained for the elastic system. Other investigations [10], have shown that the reduction in response of systems with supplemental viscous damping is highly dependent on the plan-wise distribution of the devices, implying that such reduction does not depend only on $\epsilon_{SD}$ but also on $\rho_{SD}$. Dampers in two directions and physical limits for the structural parameters were considered. The optimal plan-wise distribution of supplemental dampers has also been considered [11–13] by minimizing different response quantities.

On the other hand, the concept of weak torsional balance as introduced herein, the rule for optimal location of dampers, and the parametric study on response reduction factors are
Torsional balance is defined as a property of an asymmetric structure that leads to similar deformation demand in structural members equidistant from the geometric center (GC) of the building plan. It can be defined in a strong or a weak form. The former implies an uncoupling of the lateral and torsional motions and leads (nominally) to equal deformation demand in all structural members. The latter, which allows for rotation of the building plan, only implies an equal norm of the displacement demand on resisting planes symmetrically placed with respect to the GC. It is this weak form of torsional balance that is considered in this investigation. For the time being, it is only the effect of torsional balance on the dynamic response of an asymmetric structure that is considered. The reader may find a more extensive discussion of the idea of torsional balance with energy dissipation devices elsewhere [14].

Consider a mono-symmetric single-story building with an arbitrary location of the CM and stiffness eccentricity $e_{sx}$. The origin of the co-ordinate system is coincident with the location of the CM (Figure 1). The motion of the rigid (in-plane) diaphragm is defined by the horizontal displacements $u_y$ in the $y$-direction, and a normalized rotation $l_xu_\theta$ about a vertical $z$-axis (Figure 1). A single frictional damper is included along the $y$-axis at distance $e_d$ from the center of mass (CM). The mass is lumped at the diaphragm level and has a radius of gyration $\rho = \sqrt{(l_x^2 + l_y^2)/12} = l_x\sqrt{(1 + \alpha^2)/12}$ where $\alpha = l_y/l_x$ is the plan aspect ratio. A constant modal damping ratio $\zeta = 5\%$ was considered for the primary structure. Although this model may seem oversimplified, it is sufficient to present the theory of torsional balance and the results presented carry over to more complex structures. Furthermore, the justification for

![Figure 1. Mono-symmetric single-story model considered with stiffness eccentricity, $e_{sx}$.](image-url)
not including more dampers in plan leading to $\rho_{SD} \neq 0$ will become apparent soon in light of the concept of strong torsional balance.

Let us assume that the degrees of freedom $\mathbf{u}(t)^T = [u_x(t, e_d) \ l_x u_y(t, e_d)]$ of the structure have been computed for a known location $e_d$ of the frictional damper—the interesting point will be to look at the correlation between the translational and rotational motions of the structure as the damper moves from one position to another. In what follows, computed responses are assumed to be realizations of an ergodic random process with zero mean; the same analysis could be performed by using time-average operators but statistical moments are simpler to deal with algebraically. Therefore, the displacement (velocity or acceleration) at distance $p$ from the CM would be of the form $\tilde{u}_y(p) = u_y(t, e_d) + pu_x(t, e_d)$ and, hence, the mean square value of the displacement is $E[\tilde{u}_y(p)\tilde{u}_x(t, e_d)]^2 = E[u_y(t, e_d)^2] + 2pE[u_y(t, e_d)u_x(t, e_d)] + p^2E[u_x(t, e_d)^2]$, which represents a parabola in $p$ (Figure 2). For a given displacement profile of the building plan, the minimum value for $E[\tilde{u}_y(p)^2]$ is achieved at a point at distance $p^*$ from the CM, i.e.

$$\frac{\partial E[\tilde{u}_y(p)^2]}{\partial p} = 2E[u_yu_0] + 2pE[u_y^2] = 0, \text{ i.e., } p^* = -\frac{E[u_yu_0]}{E[u_y^2]} = -\frac{\rho_{y\theta} \sigma_y}{\sigma_\theta} \tag{1}$$

where $\rho_{y\theta}$ is the linear correlation coefficient between lateral and torsional motions at the CM and $\sigma_y, \sigma_\theta$ are the corresponding standard deviations. In the last expression of Equation (1), the damper location $e_d$ and time $t$ have been omitted for clarity but remain implicit in the computed responses. Equation (1) implies that at point $p^*$, the structure attains the smallest mean square displacement value (MSV) since the second derivative of $E[\tilde{u}_y(p)^2]$ with respect to the location $p$ is always positive.
Let us look now for a point in plan such that the correlation between the displacement \( \tilde{u}_y^{(p)} \) and plan rotation \( u_\theta \) be zero, i.e.

\[
E[\tilde{u}_y^{(p)}u_\theta] = E[u_i(t,e_d)u_\theta(t,e_d)] + pE[u_\theta(t,e_d)^2] = 0
\]

\[
p^* = -\frac{E[u_i(t)u_\theta(t)]}{E[u_\theta(t)^2]} = -\rho_{y\theta}(\sigma_y/\sigma_\theta)
\]  

(2)

Notice that the position \( p^* \) provided by Equations (1) and (2) is the same, i.e. the point in the building plan for which there is zero correlation between lateral and torsional motions and that with minimum MSV of the displacement coincide.

The point defined by the position \( p^* \) will be denoted, in the average sense presented (second moments or mean-square), as the empirical center of balance (ECB) of the building plan. Thus, at the ECB, translations and rotations are statistically uncorrelated (orthogonal in the mean-square sense) and if the ECB and CM coincide, the structure behaves, in the mean square sense, like an uncoupled system.

The single-story structure has eccentricity \( e_s \) between the CM and the center of stiffness (CS). If the plan undergoes pure rotation \( u_\theta(t) \), such rotation will occur with respect to the CS and the displacement at the CM will be \( u_y(t) = -e_s u_\theta(t) \). Substituting in Equation (2)

\[
p^* = -\frac{E[u_i(t)u_\theta(t)]}{E[u_\theta(t)^2]} = -\frac{(-e_s)E[u_\theta^2]}{E[u_\theta^2]} = e_s
\]

(3)

and, hence, for this simple single-story case the location of the ECB and the CS would coincide. A corollary of this result is that it would be possible to estimate from measured torsional motions of a structure an ‘average’ location of the CS of the building by finding the point in plan with least MSV. This result is a time counterpart of the result presented earlier by Şafak et al. [15].

**Lemma 1**

For a given structure and response, the ECB always exists and is unique.

Existence and uniqueness of the ECB for a fixed structural configuration is proved by Equation (2), with the exception of the singularity of nominally symmetric structures with \( E[u_\theta(t)^2] = 0 \), in which case the ECB is at infinity and strong torsional balance is achieved. Notice that if \( p^* > 0 \), the ECB is at the right of the CM and the correlation \( E[u_i(t)u_\theta(t)] < 0 \); the opposite occurs for \( p^* < 0 \). Thus, the correlation between the lateral displacement and plan rotation changes sign as the ECB shifts from one side to the other of the CM.

Let us turn to the question of how to balance an asymmetric building plan. Recall that the ECB at distance \( p^* \) from the CM has the minimum MSV of the displacements of the building plan, where the value is:

\[
E[\tilde{u}_y^{(p^*)^2}] = E[u_i^2] + 2 \left( -\frac{E[u_i u_\theta]}{E[u_\theta^2]} \right) E[u_i u_\theta] + \left[ -\frac{E[u_i u_\theta]}{E[u_\theta^2]} \right]^2 E[u_\theta^2]
\]

\[
E[\tilde{u}_y^{(p^*)^2}] = E[u_i^2] - \frac{E[u_i u_\theta]^2}{E[u_\theta^2]}
\]

(4)
Therefore, Equation (4) provides the value at the vertex of the parabola corresponding to the MSV displacement profile of the building plan (Figure 2). If the ECB lies within the plan, the MSV of the displacement increases quadratically toward the edges of the structure. Therefore, it is simple to state the conditions of weak torsional balance, i.e. to attain equal MSV of the displacements at both edges of the building plan.

**Lemma 2**

Weak torsional balance is achieved in an asymmetric single-story structure by optimally placing the damper at distance \( e^* \) from the CM such that the resulting ECB is at equal distance from both edges of the building plan.

To prove this claim notice that for \( \tilde{u}_y(t) \) defined at the ECB and plan rotation \( u_\theta(t) \), the MSV of the displacement at distance \( \pm d \) from the ECB is

\[
E[\tilde{u}_y(t, e^*_y)^2] = E[\tilde{u}_y^2] + 2(\pm d)E[\tilde{u}_y u_\theta] + d^2E[u_\theta^2]
\]

\[
E[\tilde{u}_y(t, e^*_y)^2] = E[\tilde{u}_y^2] + d^2E[u_\theta^2]
\]

since \( E[\tilde{u}_y u_\theta] = 0 \) at the ECB. A physical interpretation of Equation (5) says that because at the ECB the lateral and torsional motions are uncoupled in the mean-square-sense, the MSV of the displacements at the edges are a direct sum of the MSV of the lateral displacement at the ECB and the MSV of the building plan rotation multiplied by the distance squared from the ECB to the resisting plane considered. Therefore, the mean square values of the displacements of two resisting planes will be the same if and only if their distance \( d \) to the ECB is the same. Otherwise, the MSV of the displacement will always be larger at the edge or plane farthest from the ECB (Figure 2).

Defining as \( d_f \) the distance from the CM to the edge located on the positive x-axis (Figure 1), the geometrical condition for optimal placement of the damper is

\[
p^* = - \frac{E[u_y(t, e^*_y)] u_\theta(t, e^*_y)}{E[u_\theta(t, e^*_y)^2]} = d_f - \frac{l_x}{2}
\]

where \( u_y(t) \) and \( u_\theta(t) \) are defined at the CM. Equation (6) implies that if the CM of the building plan coincides with the GC, i.e. \( d_f = l_x/2 \), then \( p^* = 0 \). Only for that particular case, the condition for optimal damper placement would be the zero-correlation condition,

\[
p^* = - \frac{E[u_y u_\theta]}{E[u_\theta]} = 0 \Rightarrow E[u_y u_\theta] = 0
\]

The condition \( p^* = 0 \) also implies that the CM coincides with the ECB. Thus, a corollary of Lemma 2 is that the optimal location of the damper for \( p^* = 0 \) must be such that \( e^*_y \) counteracts the effect of the stiffness eccentricity \( e_s \) by forcing the ECB to coincide with the CM. Thus, the mirror rule proposed earlier by Goel [8, 9] has a qualitative justification in terms of the ECB. However, it should be expected that for structures where rotation of the plan is more significant, say uncoupled lateral-to-torsional frequency ratio \( \Omega_\theta \leq 1 \), one should expect some deviations with respect to the mirror rule.

It is relevant to mention that the ECB is not in general a center of stiffness, damping, or strength. Its location depends on the same parameters as the response of the structure. In that sense, Equation (2) states a general condition needed to achieve weak torsional balance.
in the structure but says nothing on how to achieve it. The latter depends primarily on the stiffness, damping properties, and location of the damper as well as the dynamic properties of the structure. In solving for $e_d^*$ from Equations (6) or (7), the unknown optimal location of the damper $e_d^*$ is implicit in the lateral and torsional response at the CM. In this investigation, the optimization toolbox of MATLAB [16] was used to find the $e_d^*$ values.

Because in the previous derivations we have not used any assumption on the response of the system, and considered $u_l(t)$ and $u_t(t)$ as available signals, which may come from an elastic or inelastic, single or multistory structure, with or without dampers, the concept of the ECB is completely general, and should include for instance the behavior of any inelastic structure with any type of damper subjected to any ground motion. Therefore, these results are the key to propose guidelines to torsionally balance a structure in the mean-square-sense presented. Just to satisfy our curiosity, Figures 3 and 4 show the inelastic earthquake response of a (0,3) structure, i.e. a structure with three resisting planes in the $y$-direction and zero in the $x$-direction, subjected to the Sylmar earthquake record. For the sake of brevity, only
Figure 4. Cross-correlation between the elastic lateral and rotational motion, standard deviation of the stiff and flexible edge response, and empirical center of balance (ECB) for a torsionally flexible ($\Omega_0 = 1.0$) and stiff ($\Omega_0 = 1.25$) structure: (a) building plan; (b) correlation; (c) stiff and flexible response; and (d) empirical center of balance.

The response histories and displacement orbits for the case $\Omega_0 = 1.25$ are shown in Figure 3; results for other $\Omega_0$ values lead to similar observations. The structure is defined by a lateral uncoupled frequency ratio $\Omega_x = \omega_x/\omega_y = 1$, normalized eccentricity $\hat{e}_x = e_x/\ell_x = 0.08$, uncoupled period in the $y$-direction $T_y = 1s$, and a plan aspect ratio $\alpha = 0.5$. Structural elements in the $y$-direction are modeled as one-dimensional Bouc–Wen elements with a total normalized capacity $\hat{F}_d_y = 0.3$; the Bouc–Wen parameters chosen are $A_o = 1$, $\beta = \gamma = 1$, $\kappa = 0.05$ and $n = 3$. The initial stiffnesses selected for the resisting planes, $k_{xy}$, are proportional to the element strengths. Moreover, the damper has normalized capacity $\hat{F}_d_y = 5\%$, initial stiffness $k_o = 10K_y$, and Bouc–Wen parameters $A_o = 1$, $\beta = \gamma = 0.5$, $n = 10$ and $\kappa = 0$. The structure without a frictional damper shows inelastic displacement offsets of the flexible edge and significant discrepancies between the peak displacements at the stiff and flexible edges of the structure. For this case, the correlation between lateral and torsional motions is $\rho_{xy} = -0.52$. By placing the damper on the flexible edge the correlation decreases to $\rho_{xy} = 0.361$, but large discrepancies are observed between the displacement histories at both edges of the structure. Finally, by
placing the frictional damper at the optimal value \( e_d^* = -0.17 l_s \), the correlation is reduced essentially to zero and the peak response at both edges is quite similar—recall that we only impose the weak balance condition implying that the MSV of the displacement at both edges be the same, which is different to imposing identical peak values.

The results shown in Figure 4 for two inelastic structures \( \Omega_0 = 1 \) and 1.25 validate the general ideas of the ECB presented in this section. The analysis shows that the optimal damper eccentricities are for these structures \( e_d^* = 0.03 l_s \) and \( -0.17 l_s \), respectively. These eccentricities are such that they locate the ECB on top of the CM of the structure. For this damper location, the standard deviations of the response at the stiff and flexible edges are the same and the correlation between lateral and torsional motions are zero. Thus, the conditions imposed by Equations (6) or (7) are still valid for this inelastic case. Recent results also show that the same would happen if we select the motions of an arbitrary inelastic multistory structure, showing that the stated condition to define torsional balance (but not the value) remains invariant to the nature of the structure that generates the response.

Notice that the condition \( p^* = 0 \) only implies the location of the ECB, i.e. the position of the vertex of the parabola of mean square displacement values across the building plan. It is apparent that if we could reduce the curvature of such parabola (Figure 2), not only will the edge displacements be identical in the mean square sense but also the displacements of all resisting planes in the building. This is the condition of strong torsional balance and is derived by minimizing \( E[\sigma^2] \) in Equation (5). It is then possible to move in the trajectory from weak to strong torsional balance by increasing \( \rho_{SD} \) while preserving the location of the ECB. Thus, \( \rho_{SD} \) controls \( E[\sigma^2] \) and should always be maximized for a given damper capacity by placing the dampers, symmetrically, but as far as possible from the ECB.

**GROUND MOTIONS AND RESPONSES CONSIDERED IN THE ANALYSIS**

In this investigation, three artificial subduction type earthquake records compatible with the design spectrum of the Chilean code for seismic isolation [17] and five impulsive ground motions were considered. The artificial motions correspond to a design spectrum with 10% probability of exceedance in 50 years and stiff soil condition (shear wave velocity, \( 400 \text{ m/s} < V_s < 900 \text{ m/s} \)); the impulsive earthquake records are: Arleta, Sylmar, Newhall (1994), Corralitos (1989), and Kobe (1995). A stochastic analysis was also performed with the subduction-type ground motions in order to extend the results and obtain smoother trends.

Results are presented for mono-symmetric structures \( \hat{e}_x \neq 0, \hat{e}_y = 0 \), torsional-to-lateral frequency ratios \( \Omega_0 = 0.8, 1 \), and 1.2, plan aspect ratio \( \alpha = 0.5 \), and damping ratio \( \xi = 5\% \) in the primary system. Besides, three building periods are considered in the analysis \( T_y = 0.5 \text{ s}, 1 \text{ s}, \text{ and } 2 \text{ s} \). Normalized capacities of the frictional devices are \( \beta = 2.5\%, 5\%, 10\% \) and \( 5\%, 10\%, 20\% \) for the non-impulsive and impulsive ground motions, respectively. An elastoplastic model with large initial stiffness is used to characterize the constitutive behavior of the frictional damper.

The analytical responses considered are the mean peak normalized displacements at the stiff and flexible edge of the building plan, denoted as \( \hat{u}_t = u_t^{(d)} / u_t \) and \( \hat{u}_f = u_f^{(d)} / u_t \), respectively, where the supraindex \( (d) \) denotes the structure with frictional dampers, and in the denominator the peak response of the corresponding edges of the structure without dampers, unless stated otherwise. When the same responses are normalized with respect to the symmetric counterpart,
the symbol $\hat{u}_o$ is used. Results are also presented in terms of total displacements and the torsional index $T_i$. This torsional index is defined as the ratio between the maximum absolute difference between the lateral displacements at the flexible and stiff edges over the displacement $u_o$ at the CM of the same structure, i.e. $T_i = \max |u_i - u_i|/u_o$. Most response results are presented for two locations of the damper defined by (i) the optimal location based on torsional balance corresponding to the solution of Equation (6), and (ii) the response provided by the mirror rule proposed earlier [8, 9].

ELASTIC RESPONSE OF THE PRIMARY SYSTEM

Figure 5 shows the mean optimal location $e^*_d$ of the damper for the artificial ground motions as a function of the normalized stiffness eccentricity $\hat{e}_{sx}$ for three different values of the torsional-to-lateral frequency ratio $\Omega_\theta = 0.8$, 1, and 1.2. The optimal values presented for $e^*_d$ are associated with the torsional balance criteria (Equation (6)) and computed by the true inelastic model (solid line) of the damper and its linear equivalent statistical model [18] (dashed line). Each row of plots corresponds, from top to bottom, to uncoupled periods $T_y = 0.5$, 1, and 2 s, respectively, and normalized stiffness eccentricities $\hat{e}_{sx}$ range from 0 to 0.25. Several observations are obtained from this figure. In most cases, linearized responses lead to similar optimal values $e^*_d$ as those obtained by the inelastic analysis of the structure. Therefore, at least for design purposes, results from a linearized statistical approximation of the system seems to provide reasonably accurate trends and values of $e^*_d$. As the static eccentricity $\hat{e}_{sx}$ increases, the eccentricity of the damper also tends to increase but with the opposite algebraic sign. This is consistent with the trend of the mirror rule to locate the damper [8]; such a rule would be represented in this plot as a straight line with slope of $-1$. However, it is apparent that for the shorter structural periods, torsionally flexible structures $\Omega_\theta = 0.8$, and small static eccentricity values, say $\hat{e}_{sx} < 0.1$, the optimal location of the damper is positive, i.e. on the same side as the static eccentricity of the system. Therefore, in those cases the mirror rule would not work properly. As the capacity of the dampers increases, the slopes of the optimal eccentricity curves decrease. This is consistent with the physical intuition that a small change in the location of a large capacity damper will have a greater impact in the torsional response of the structure. On the other hand, small normalized frictional capacities, such as $\beta = 2.5\%$ are not able to control the lateral-torsional response in the more rigid structures, $T_y = 0.5$ s, with large eccentricities $\hat{e}_{sx} > 0.05$. As the structure becomes more flexible, almost any capacity in the damper is able to weakly balance its torsional response. Finally, please note that the trends of $e^*_d$ become more closely linear as the lateral flexibility and torsional stiffness of the structure increases; therefore, the mirror rule provides a rough average estimate (although very simple) of the true optimal values for longer periods and larger $\Omega_\theta$.

A comparison between the normalized displacements, reduction factors, and torsional index as computed from the optimal balance and mirror criteria are presented in Figures 6 and 7 for structures with uncoupled lateral periods $T_y = 0.5$ s and $T_y = 1$ s. The optimal values of $\hat{e}_{dx}$ used in the analysis correspond to the linearized stochastic approximation, which are certainly not exact for a given record, but sufficiently accurate for design purposes. Results for two frictional capacities, 2.5% and 10%, and two torsional-to-lateral frequency ratios $\Omega_\theta = 0.8$ and 1.2 are included in the figure. The first row of plots shows the stiff and flexible edge displacements normalized with respect to the peak response of the nominally symmetric case,
Figure 5. Mean results of the optimal FD location for subduction type ground motions as a function of normalized static eccentricity for $\Omega_0 = 0.8$, 1.0, 1.2 and uncoupled lateral period, $T_y = 0.5$, 1.0 and 2.0 s.
the second row, the torsional index of the structure defined as the peak difference of the edge displacements over the peak response at the CM, and the third row, the reduction factors \( \hat{u}_r \) and \( \hat{u}_f \) obtained as the ratio between the same peak edge response of the structure with and without the damper.

As shown in Figure 6 for \( T_y = 0.5 \) s, the peak edge displacements corresponding to the optimal criteria are quite similar. Coincident solid and dashed lines in this figure imply that the structure behaves in the MSV sense like a nominally symmetric system. Although such should always be the case by the definition of \( e^*_d \), this cannot be assured by the solution presented since the optimal damper location values used, \( \tilde{e}_d^* \), were derived from an equal variance criteria and an average power spectrum density function (PSD), not from the individual records. This implies that the true optimal value of \( e^*_d \), for each record and structure, may be different from the \( \tilde{e}_d^* \) value selected to compute the results presented in this figure. This was done with the objective of using an optimal rule for \( e^*_d \) that is less sensitive to the record used. By comparing the mirror (thin trace) and optimal criteria (thick trace), it is apparent that the former may lead in this case to displacement demands on the flexible and stiff edge that are significantly different. The torsional index also tends to be smaller when using the optimal \( \tilde{e}_d^* \) values, especially for larger eccentricities and smaller damper capacity (\( \beta = 2.5\% \)). In terms of reduction factors, the use of \( \tilde{e}_d^* \) leads to larger response reduction factors than for the mirror criteria with the exception of \( \beta = 2.5\% \) and \( \Omega_d = 1.2 \). In that case, the stiff edge of the structure shows an amplification with respect to the same displacement in the structure without the damper. In spite of this larger amplification, the displacement demand on both edges is similar. This fact shows that by using response reduction factors as a single criteria to select damper characteristics in a structure may be misleading.

Analogous results for the \( T_y = 1 \) s structures are presented in Figure 7. Although better than the mirror rule, the normalized edge displacements based on the optimal criteria present larger discrepancies than for the \( T_y = 0.5 \) s structures. These results do not imply a contradiction with the weak torsional balancing concept, but they show that the simplified average procedure proposed in terms of optimal eccentricities \( \tilde{e}_d^* \) may still lead to some discrepancies between the response at both edges, particularly for static eccentricities \( \tilde{e}_{ss} > 0.1 \). The earthquake response of the structure is that complex and variable that it is difficult to propose accurate globally applicable rules. Moreover, it is observed that for most structures with damper capacity \( \beta = 0.1 \), the response reduction factors achieved at both edges relative to the structure without dampers are significant, say \( 1/\hat{u} = 1/0.6 = 1.7 \). Results also show that by using small frictional capacity when dealing with torsionally flexible structures, the responses of the stiff and flexible edges may be amplified by 50\% and 100\% by the use of the damper; however, the resulting design is still balanced. A comparison of the torsional indices obtained by the mirror and optimal criteria show that similar values of plan rotation should be expected by these rules for \( \beta = 0.025 \) and slightly smaller for the optimal criteria with \( \beta = 0.1 \).

Let us consider now the earthquake response of structures with FD subjected to impulsive ground motions. Optimal average damper eccentricities for the five impulsive ground motions considered are presented in Figure 8 as a function of the normalized eccentricity \( \tilde{e}_{ss} \), three values of the torsional-to-lateral frequency ratio \( \Omega_d = 0.8 \), 1, and 1.2, three uncoupled structural periods \( T_y = 0.5 \), 1, and 2 s, and three normalized frictional damper capacities \( \beta = 5\% \), 10\%, and 20\%; larger capacities are required in this case as opposed to subduction type motions to achieve optimal responses. The optimal values presented for \( e^*_d \) correspond to average values obtained for the five records and are associated with the optimal torsional balance
Figure 6. Comparison of normalized displacement, torsional index, and displacement reduction factors for stiff structures ($T_y = 0.5\text{s}$) with FD capacities 2.5% and 10%, subjected to subduction type ground motions: (a) $\beta = 2.5\%$; and (b) $\beta = 10\%$. 

Figure 7. Comparison of normalized displacement, torsional index, and displacement reduction factors for intermediate period structures ($T_i = 1.0\text{ s}$) with FD capacities 2.5% and 10%, subjected to subduction type ground motions: (a) $\beta = 2.5\%$; and (b) $\beta = 10\%$. 

Figure 8. Mean results of the optimal FD location for impulsive type ground motions as a function of normalized static eccentricity for $\beta = 5\%$, $10\%$, and $20\%$.
criteria (Equation (6)) and computed by the true inelastic model of the FD. Again, stiffness eccentricities \( \hat{\epsilon}_{sx} \) vary from 0 to 0.25. The trends observed in \( e^*_d \) are similar to those obtained for subduction type motions; naturally, there exist values of \( \hat{\epsilon}_{sx} \) for which certain singularities in the response are observed. In general, as the static eccentricity \( \hat{\epsilon}_{sx} \) increases, the optimal eccentricity of the damper also increases but to the opposite side with respect to the CM. This is again consistent with the trend inferred by the mirror rule. It is apparent again that for the shorter structural periods \( T_y \leq 1 \) s, torsionally flexible structures \( \Omega_\theta = 0.8 \), and small static eccentricity values, say \( \hat{\epsilon}_{sx} < 0.05 \), the optimal location of the damper tends to be positive, i.e. on the same side as the static eccentricity of the system. In those cases the mirror rule would fail to predict the right location of the damper. Also, as the structure becomes more flexible and dampers are stronger (\( \beta > 10\% \)), a very simple rule would be to locate the damper close to the CM of the structure. Besides, as the capacity of the dampers increases, the slopes of the optimal eccentricity curves decrease in absolute value. Thus, a general design criteria for optimal damper location must include the damper capacity as a parameter. As stated before, this is consistent with the physical intuition which shows that for larger capacities, small variations in the damper location enable us to torsionally balance the building plan. On the other hand, torsionally stiffer structures require larger frictional capacities to balance the plan in the weak sense and, hence, optimal damper eccentricities are also a function of the period of the structure and the torsional-to-lateral frequency ratio. Finally, if Figures 5 and 8 are superimposed, there exist remarkable similarities in the trends despite the large discrepancies between the characteristics of the records considered.

Figures 9 and 10 show a comparison between the normalized edge displacements, reduction factors, and torsional index as computed from the optimal balance and mirror rule for structures with uncoupled lateral periods \( T_y = 0.5 \) s and \( T_y = 1 \) s. These results are counterparts of the results presented in Figures 6 and 7, but in this case the optimal average values of \( e^*_d \) presented in Figure 8 are used to compute the response to all building records. Indeed, the true optimal values of \( e^*_d \) for each record have not been used in the analysis in order to prevent using optimal damper eccentricities that are record dependent; nevertheless, this can always be done for a specific design of a structure. Results for normalized frictional capacities \( \beta = 5\% \) and 20\% and two torsional-to-lateral frequency ratios \( \Omega_\theta = 0.8 \) and 1.2 are considered in both figures. As before, the first row of plots shows the stiff and flexible normalized edge displacements, the second row shows the torsional index, and the third row, the reduction factors \( \hat{\nu}_I \) and \( \hat{\nu}_R \).

For \( T_y = 0.5 \) s and static eccentricities \( \hat{\epsilon}_{sx} < 0.20 \) (Figure 9), the peak displacements at both edges computed with the optimal criteria are similar. The discrepancies observed in their mean values are due to the variability in response caused by the different ground motions. For this range of eccentricities, torsional amplifications relative to the symmetric case are less than 40\% and reduction factors relative to the asymmetric response of the structure without dampers range between, say, 2 and 0.7. Therefore, though in most cases the use of FD leads to response reduction factors larger than 1, there exist cases, such as for \( \hat{\epsilon}_{sx} > 0.30 \) or torsionally stiff structures \( \Omega_\theta = 1.2 \) with small damper capacity, where the amplifications in the stiff edge may be larger than for the structure without dampers. However, reduction factors are misleading since the lateral displacement at the building edges of the structure with dampers is still more balanced than the response of the original asymmetric structure without dampers. In this particular case, the mirror criteria for placing the dampers leads to torsionally balanced designs that are in agreement with those of the optimal criteria and in some cases even with smaller average variability in the response of the stiff and flexible edges. Finally, torsional
Figure 9. Comparison of normalized displacement, torsional index, and displacement reduction factors for stiff structures ($T_y = 0.5 s$) with FD of capacities 5% and 20%, subjected to impulsive ground motions: (a) $\beta = 5\%$; and (b) $\beta = 20\%$. 

Figure 10. Comparison of normalized displacement, torsional index, and displacement reduction factors for intermediate period structures ($T_f = 1.0$ s) with FD of capacities 5% and 20%, subjected to impulsive ground motions: (a) $F = 5\%$; and (b) $F = 20\%$. 

indices increase with the eccentricity of the structure reaching values of about 3 or 4 for $\hat{e}_{ss} > 0.1$; they also tend to increase as the frictional capacity increases.

Analogous results for structures with uncoupled lateral period $T_y = 1\, s$ are presented in Figure 10. For this case the optimal criteria lead to torsional designs that are quite balanced, especially in the case of larger frictional capacity. In general, normalized edge displacements computed with the optimal criteria present smaller discrepancies than those obtained with the mirror criteria. Besides, torsional amplifications due to plan asymmetry are bound to less than 30% and in many cases less than that. Torsional indices increase with static eccentricity and reach stable values of about 2 to 4 for $\hat{e}_{ss} > 0.1$. Furthermore, response reduction factors are for most cases larger than 1, which implies that the use of FD in the structure simultaneously reduces the displacement demand at both edges of the plan. Results for other more flexible structures as well as with other damper capacities follow similar trends [19].

As stated above, once the structure is torsionally balanced in the weak sense presented earlier by using single or multiple friction dampers, additional dampers should always be placed as far as possible and in a symmetric layout with respect to the ECB. The symmetric layout implies that the ECB remains at the same location, and the standard deviation of the response at both edges will remain essentially identical. Therefore, supplemental dampers may be used to reduce the MSV of the rotation further while keeping the mean-square uncoupling achieved between translational and rotational motions. Consequently, from a design standpoint, the first step is to achieve weak torsional balance by placing the ECB at equal distance from both edges. Secondly, to move in the direction of strong torsional balance by maximizing $\rho_{SD}$.

CONCLUSIONS

In this article, the concept of torsional balance of simple structures with frictional dampers was evaluated emphasizing the optimal plan location of dampers. Torsional balance was defined in an average sense by minimizing the correlation between the translation and rotation of the building plan. An explicit equation to achieve this condition was presented. Results obtained recently show that this equation may be used in the case of linear, non-linear, single, or multistory structures. It provides a common ground to evaluate different techniques to achieve torsional balance in an asymmetric structure. The specific case of frictional dampers was evaluated in this research. It is concluded that, in addition to the reduction in response due to the supplemental damping provided by the dampers, the designer will be capable of controlling the torsional response of the structure by placing the dampers in a position such that the ECB of the structure is at equal distance relative to both edges of the building plan. For geometrically regular plans, this implies making the ECB coincident with the GC of the plan. A large number of optimal responses showed that the criteria of placing the damper according to the mirror rule, although reasonable, is less effective than using the optimal location. However, it is recognized that the optimal criteria is more difficult to cast into simple expressions as opposed to the simplicity of the mirror rule. For flexible structures $T_y > 2\, s$, an approximate optimal location of the damper is the CM. Although the frictional capacities required are considerably different, optimal damper eccentricities for structures subjected to subduction and impulsive ground motions show similar trends. By using the optimal criteria, the maximum displacements at both edges of the building plan are always similar and less
than twice the response of the nominally symmetric counterpart. Although this factor of two may seem large, it can be controlled in many cases by adding more frictional capacity. Indeed, the optimal design of a structure with frictional dampers is achieved in two stages: (i) place the ECB as stated above, and (ii) reduce the torsional motions further by adding dampers that preserve the location of the ECB, but are as far as possible from each other in plan, hence, maximizing $\rho_{sd}$.

ACKNOWLEDGEMENTS

This investigation was funded by the Chilean National Fund for Science and Technology, Fondecyt through Grant #1020774. The authors are grateful for this support. The authors also thank the anonymous contributions of the two reviewers of this article.

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