Accidental torsion due to overturning in nominally symmetric structures isolated with the FPS

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SUMMARY

Overturning of a structure causes variations in the normal loads of the isolators supporting that structure. For frictional isolators, such variation leads to changes in the frictional forces developed and, hence, in the strength distribution in plan. For frictional pendulum system (FPS) isolators, it also causes changes in the pendular action, i.e. in the stiffness distribution of the isolation interface. Therefore, although the structure is nominally symmetric it develops lateral—torsional coupling when it is subjected to two horizontal components of ground motion. This coupling is denoted herein as accidental torsion due to overturning, and its effect in the earthquake response of nominally symmetric structures is evaluated. Several parameters are identified to control this coupling, but the most important are the slenderness of the structure and the aspect ratio of the building plan. Results are presented in terms of the torsional amplification of the deformations of the isolation base and the interstorey deformations of the superstructure. The FPS system is modelled accurately by including true large deformations and the potential uplift and impact of the isolators. Impulsive as well as subduction-type ground motions are considered in the analysis, but results show small differences between them. An upper bound for the mean-plus-one standard deviation values of the torsional amplifications for the base due to this accidental torsion is 5%. This implies that for design purposes of the isolation system such increase in deformations could probably be neglected. However, the same amplification for the interstorey deformations may be as large as 50%, depending on the torsional stiffness and slenderness of the superstructure, and should be considered in design. In general, such amplification of deformations decreases for torsionally stiffer structures and smaller height-to-base aspect ratios. Copyright © 2003 John Wiley & Sons, Ltd.

INTRODUCTION

Accidental torsion in nominally symmetric structures isolated with the frictional pendulum system (FPS) is due to factors such as unforeseen variabilities in the geometric and frictional properties of the isolation system, uncertainties in the stiffness and mass distribution of the superstructure, torsional input motions, and variabilities in the normal isolator forces as a result of the overturning of the structure. Although this latter source of lateral—torsional coupling

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of the structure can be predicted analytically, it has been classified as accidental only because conventional small deformation models do not account for it properly.

Current design codes for seismically isolated structures do not account explicitly for the increase in response of the isolation and structure due to accidental torsion. Moreover, they do not distinguish between lateral–torsional coupling effects coming from different isolation solutions. For instance, only as an example, structures isolated with rubber bearings are far less sensitive to variabilities in the normal isolator forces than structures isolated with the FPS. Therefore, in rubber isolation, overturning of the superstructure will not lead to significant stiffness and strength asymmetries as it does for the FPS case. On the other hand, structures with rubber isolators will be more sensitive to asymmetries in the distribution of mass of the structure than structures with the FPS, which tend to control such asymmetry naturally by aligning the centres of stiffness and frictional resistance of the isolation system with the CM of the superstructure [1, 2].

It was shown earlier that the FPS is capable of controlling the torsional response of structures with asymmetries in the mass distribution [1, 2]. However, these results were limited to single-storey systems subject to unidirectional ground motions. That the ground motion is unidirectional is important since overturning of the superstructure leads to lateral–torsional coupling only if the ground motion is bidirectional. More recently, the earthquake response of nominally symmetric and asymmetric structures with the FPS has been studied experimentally by using a three-storey structural model and a three-dimensional shaking table [3].

Since accidental torsion will usually lead to increases in response that are moderate, the structural model of the isolation system should be consistent with such level of accuracy. Therefore, although small deformation models [1, 2, 4, 5] lead to reasonably accurate predictions of global responses in moderate earthquakes, they lead to significant errors in base shear, interstorey deformations, member deformations and forces, and normal isolator forces, particularly for impulsive ground motions causing uplift and impact. Consequently, a true large deformation model of the FPS will be considered in the analysis [6–8].

This study of accidental torsion in structures isolated with the FPS has the following objectives: (i) to understand the variables that control accidental torsion due to overturning; (ii) to evaluate the expected increase in response that can attributed to accidental torsion and its dependence with the building parameters, (iii) to evaluate the efficacy of the FPS to control accidental torsion as opposed to conventional structures, and (iv) to propose reliable bounds to account for accidental torsion in building design.

**STRUCTURAL SYSTEM CONSIDERED**

Nominally symmetric structures were selected because they are the most sensitive to accidental torsion [9]. On one hand, they enable us to study accidental torsion effects without contamination of natural torsion, and, on the other, nominally symmetric structures lead to an upper bound for the increase in edge deformations and member forces due to accidental torsion [9]. Therefore, the superstructures considered are symmetric \( n \)-storey buildings with rectangular plan of aspect ratio \( \gamma = \frac{b}{a} \) (Figure 1), where \( a \) and \( b \) are the plan dimensions in the \( X \) and \( Y \) directions, respectively. The slenderness ratio of the building is defined as \( \beta = \frac{H}{a} \), where \( H \) is the height of the building above the isolation base. All storeys are assumed to have the same height, and identical location of the CM of each floor.
In order to focus in the main sources of accidental torsion, the model of the isolated superstructure has been simplified by considering it as a rigid body with three degrees of freedom, the vertical motion and the rotations about the $X$- and $Y$-axes. Furthermore, each building floor is considered rigid in and out of his plane, with three degrees of freedom, the two horizontal displacements and a rotation about the vertical axis. There are several reasons that justify these modelling assumptions. First, the behaviour of base isolated structures is dominated by the behaviour of the isolation interface and not as much by the details of the model of the superstructure. Second, a larger number of parameters would preclude us to observe the general trends of the lateral–torsional coupling due to accidental factors. Third, by neglecting the vertical flexibility of the superstructure, only the high frequency motions
of the superstructure are neglected, which have little impact in the horizontal behaviour. And fourth, the results presented are validated with results obtained from true three-dimensional structural models [10].

In contrast with the rather coarse model of the superstructure, the isolation system needs to include the true large deformation effects, lateral-vertical coupling of the isolators [6–8], the vertical component of ground motion, sticking and potential uplift and impact of the isolators, especially for impulsive ground motions. These effects are particularly important in computing local responses of the structure, such as interstorey deformations or member forces.

For an n-storey structure, the degrees of freedom are partitioned in blocks and denoted as \( X_{n+6 \times 1} = [r^T \; q^T \; u^T]^T \), where the vector \( r = [q_x, q_y, q_z]^T \) contains the rigid body motions of the superstructure relative to the ground, with \( q_z \) the vertical displacement, and \( q_x, q_y, q_\phi \), the rotations about the \( Y \)- and \( X \)-axes, respectively; the vector \( q = [x, \phi_r, \phi_t]^T \) of displacements of the isolation diaphragm relative to the ground; and the vector of floor deformations relative to the isolation base \( u_{n+1 \times 1} = [u^{(1)}; \ldots ; u^{(n)}] \), where \( u^{(j)} = [u_x^{(j)}, u_y^{(j)}, u_z^{(j)}]^T \) are the dynamic deformations of the \( j \)-th floor, i.e. excluding the floor displacements \( \phi, h_j \) and \( -\phi, h_j \) attributed to the rigid body rotations of the superstructure (Figure 1(b)).

The block diagonal mass and stiffness matrices for the structural model considered are

\[
M_s = \begin{bmatrix}
0 & 0 & \ldots & 0 \\
0 & m_2 & \ldots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \ldots & m_n
\end{bmatrix}, \quad K_s = \begin{bmatrix}
k_1 + k_2 & -k_2 & \ldots & 0 & 0 \\
-k_2 & k_2 + k_3 & \ldots & 0 & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & \ldots & k_{n-1} + k_n & -k_n \\
0 & 0 & \ldots & -k_n & k_n
\end{bmatrix}
\]

where the mass and stiffness sub-matrices \( m_j \) and \( k_j \) are

\[
m_j = m_j \begin{bmatrix}
1 & 0 & -e^{(j)}_{my} \\
0 & 1 & e^{(j)}_{mx} \\
-e^{(j)}_{my} & e^{(j)}_{mx} & \rho_j^2
\end{bmatrix}, \quad k_j = k_j \begin{bmatrix}
1 & 0 & -e^{(j)}_{sy} \\
0 & 1 & e^{(j)}_{sx} \\
-e^{(j)}_{sy} & e^{(j)}_{sx} & r_j^2
\end{bmatrix}, \quad j = 1 : n
\]

where the index \( j \) denotes the \( j \)-th floor; \( m_j \) and \( k_j \) are the mass and stiffness of the \( j \)-th floor in both directions; \( \rho_j \) and \( r_j \) are the radius of gyration of the mass and torsional stiffness with respect to the vertical axis \( Z \) passing through the geometric centre; \( e^{(j)}_{mx} \) and \( e^{(j)}_{my} \) are the mass eccentricities in the \( X \)- and \( Y \)-directions, respectively; and \( e^{(j)}_{sx} \) and \( e^{(j)}_{sy} \) are the corresponding stiffness eccentricities. For the nominally symmetric case considered in this study these eccentricities are all zero; it is also assumed that the floor masses and storey stiffnesses are all equal. Furthermore, a classical damping matrix \( C_s \) has been assumed for the superstructure. This matrix leads to a constant modal damping ratio \( \xi_s = 0.05 \) and was constructed by using the vibration modes of the superstructure with fixed base [10].

Consequently, the parameters that control the dynamic behaviour of the superstructure with fixed base are: (i) the fundamental vibration period \( T_s = 2\pi/\omega_s \); (ii) the torsional to lateral frequency ratio \( \Omega_s = \omega_{\phi_0}/\omega_s \), and (iii) the modal damping ratio \( \xi_s \).
NOMINALLY SYMMETRIC STRUCTURES ISOLATED WITH THE FPS

On the other hand, the FPS are arranged in a regular grid with $p_x$ and $p_y$ isolators in the $X$- and $Y$-directions, respectively, i.e. a total of $p = p_x p_y$ isolators (Figure 1(c)). The isolators are arranged symmetrically with respect to the CM of the structure and in downward position, i.e. the sliding surface is below the slider. The mass of the isolation diaphragm is identical to the mass of one floor, i.e. $m_b = m_0 = m$. Besides, the radius of curvature $R_0$ and the frictional coefficient $\mu$ of all isolators are assumed equal. Furthermore, to identify accidental torsion caused only by the variability of the normal isolator loads, the value of the coefficient of friction was assumed equal to $\mu = 0.07$, independent of the deformation velocity and contact pressure [11].

The restoring forces of the isolators are evaluated by the physical model of the FPS presented earlier [8], which represents the contact between the slider and the sliding surface by a gap element. The stiffness $s_k$ adopted for each of these elements is assumed proportional to the normalized vertical load carried by the isolator $w_k$, $s_k = \omega_k^2 w_k / g$, where $g$ is the acceleration of gravity and $\omega_k = 100\pi \text{rad/s (50 Hz)}$ is a reference vertical frequency associated with the gap elements. In this case, the vertical flexibility of the gap elements is enough to accommodate the different vertical deformations occurring in the different isolators as a result of the rotation in plan of the structure.

EQUATIONS OF MOTION

The equations of motion presented next correspond to the most general case including plan asymmetry. For the sake of simplicity, these equations are presented in partitioned form for the degrees of freedom $r$, $q$, and $u$ (Figure 1). The dynamic equilibrium of the superstructure in the vertical direction $Z$ and rotation about the horizontal axes $Y$ and $X$ leads to

$$\sum_{j=0}^n m_j \left( \ddot{u}_{gz} + \dot{q}_z - \dot{\phi}_x e_{mx}^{(j)} + \dot{\phi}_y e_{my}^{(j)} \right) + W_i - \sum_{k=1}^p f_{z_k} = 0 \quad (3a)$$

$$\begin{align*}
\left( \sum_{j=0}^n I_Y^{(j)} \right) \ddot{\phi}_y + \sum_{j=0}^n m_j \left( \ddot{u}_{gy} + \dot{q}_y - \dot{\phi}_x h_{j} - \ddot{\phi}_y \right) h_j - \sum_{j=0}^n m_j z^{(j)} e_{mx}^{(j)} + \sum_{k=1}^p f_{z_k} x_k = 0 \quad (3b) \\
\left( \sum_{j=0}^n I_X^{(j)} \right) \ddot{\phi}_x - \sum_{j=0}^n m_j \left( \ddot{u}_{gx} + \dot{q}_x - \dot{\phi}_x h_{j} - \ddot{\phi}_x \right) h_j + \sum_{j=0}^n m_j z^{(j)} e_{my}^{(j)} - \sum_{k=1}^p f_{z_k} y_k = 0 \quad (3c)
\end{align*}$$

where $m_i$ and $W_i$ are the mass and total weight of the structure, respectively; $\ddot{x}_j$, $\ddot{y}_j$, and $\ddot{z}_j$ are the total accelerations of the $j$th floor in the $X$-, $Y$- and $Z$-directions, respectively; $\ddot{u}_{gz}$, $\ddot{u}_{gy}$, and $\ddot{u}_{gx}$ are the ground accelerations; $I_Y^{(j)} = m_j a^2 / 12$ and $I_X^{(j)} = m_j b^2 / 12$ are the inertias of the $j$th floor with respect to the horizontal axes $Y$ and $X$; $h_j = j h_s$ is the height of the $j$th floor relative to the isolation base; $f_{z_k}$ is the vertical component of the $k$th-isolator restoring force, and $x_k, y_k$ are the horizontal co-ordinates of the isolator. While there is no uplift of the structure, the rotations $\phi_y$ and $\phi_x$ are due only to the axial deformations of the gap elements.
that model the FPS and, hence, very small. The values of the rotations $\varphi_y$ and $\varphi_z$, and the coupling with the lateral motions increase suddenly when the structure undergoes uplift.

The lateral motion of the isolation base is governed by the equation

$$m_0 \left( \mathbf{L}_w^0 \mathbf{w} + \mathbf{q} \right) + \sum_{j=1}^{n} m_j \left( \mathbf{L}_w^j \mathbf{w} + \mathbf{q} + \mathbf{u}^{(j)} + \mathbf{H}_r^{(j)} \mathbf{r} \right) + F_q = 0 \quad (4a)$$

and the lateral motion of the $j$th floor,

$$m_j \left( \mathbf{L}_w^j \mathbf{w} + \mathbf{q} + \mathbf{u}^{(j)} + \mathbf{H}_r^{(j)} \mathbf{r} \right) + f_l^{(j)} = 0, \quad j = 1 : n \quad (4b)$$

where

$$\mathbf{w} = [\ddot{u}_g, \dot{u}_g, \ddot{u}_z + g]^T$$

is the three-component input vector; $\mathbf{L}_w^0 = \mathbf{L}_w^1 = [1 \ 0 \ 0; 0 \ 1 \ 0; 0 \ 0 \ 0]$ is the influence matrix between the input and the lateral degrees of freedom; $\mathbf{H}_r^{(j)} = [0 h_j 0; 0 0 -h_j; 0 0 0]$ is the overturning matrix associated with the $j$th floor; $f_l^{(j)}$ is the total linear restoring force of the $j$th storey; $\mathbf{V}_q = [V_x, V_y, V_\theta]^T$ is the base shear and torque transmitted by the superstructure to the isolation; and $\mathbf{F}_q = [F_x, F_y, F_\theta]^T$ is the restoring shear and torque generated by the isolation system. A more condensed representation of the complete set of coupled equations of motion (Equations (2)–(4)) is shown in Appendix A. The integration of these equations of motion, which are stiff in nature, is performed by using the Runge-Kutta method of order 4. Because the effect of sticking in the isolators in the peak structural response is usually minimal [8] and it increases the computational cost drastically due to the episodes of stiffness of the equations of motion, sticking in the isolators was ignored in this study of accidental torsion. Moreover, since the computation of the response of the superstructure requires a smaller number of degrees of freedom, a dynamic reduction of order was performed (Appendix A).

**LATERAL–TORSIONAL COUPLING OF THE ISOLATION SYSTEM**

Although the inelastic restoring force $F_q$ will be computed by using Equation (A5), an alternative procedure is presented herein with the purpose of illustrating the parameters that control accidental torsion in structures isolated with the FPS. It can be shown that the restoring force of the FPS system may be written as (Appendix B):

$$F_q = \mathbf{K}_b \mathbf{q} + \mathbf{C}_b \dot{\mathbf{q}} \quad (5)$$

with

$$\mathbf{K}_b = \begin{bmatrix} k_b & 0 & -k_b e_{py} \\ 0 & k_b & k_b e_{px} \\ -k_b e_{py} & k_b e_{px} & k_b \mu_p^2 \end{bmatrix}, \quad \mathbf{C}_b = \begin{bmatrix} c_b & 0 & -c_b e_{\mu_y} \\ 0 & c_b & c_b e_{\mu_x} \\ -c_b e_{\mu_y} & c_b e_{\mu_x} & c_b \mu_p^2 \end{bmatrix} \quad (6)$$

$$k_b = \frac{1}{R_0} \sum_k N_k, \quad c_b = \sum_k \zeta_k \quad (7)$$

where $K_b$ and $C_b$ are the secant stiffness and damping matrices associated with the pendular action (stiffness) and the frictional component (dissipation), respectively; $k_b$ and $c_b$ are the lateral stiffness and damping coefficients, in which $\zeta_k = \mu N_k / \|\dot{x}_k\|$ is the equivalent damping coefficient provided by the $k$th isolator; $e_{px}$, $e_{py}$ and $\rho_p$, are the eccentricities and radius of gyration of the pendular component (stiffness), while $e_{ux}$, $e_{uy}$ and $\rho_u$ are the corresponding eccentricities and radius of gyration of the frictional component (damping). The reader may find contradictory that the frictional component of the force, which is hysteretic, i.e. independent of velocity, be represented by a product of the viscous form $C_b \dot{q}$. However, each component of the matrix $C_b$ is inversely proportional to the isolator velocity and, hence, the product $C_b \dot{q}$ results independent of the velocity $\dot{q}$. Notice that eight parameters define the system matrices $K_b$ and $C_b$ (Equations (6)–(10)). The eccentricities defined by Equations (8) and (9) caused by overturning of the superstructure are responsible of accidental torsion.

The nominal values of the system parameters associated with the pendular action are obtained by replacing the vertical loads $w_k$ for the variable normal loads $N_k$ in Equations (7)–(10), i.e.

$$
\bar{k}_b = \frac{W_t}{R_0}, \quad \bar{e}_{px} = \frac{\sum_k w_k x_k}{W_t}, \quad \bar{e}_{py} = \frac{\sum_k w_k y_k}{W_t}, \quad \bar{\rho}_p^2 = \frac{\sum_k w_k (x_k^2 + y_k^2)}{W_t},
$$

where $\bar{k}_b$ is the nominal value of the lateral stiffness of the isolation system; $\bar{e}_{px}$ and $\bar{e}_{py}$ are the nominal pendular eccentricities, which value is zero for nominally symmetric structures; and $\bar{\rho}_p$ is the nominal radius of gyration. Since the input is a zero-mean process, these nominal values for the parameters correspond to the mean values in time of the instantaneous values of the corresponding parameters. As it will be shown later, $\bar{\rho}_p$ is an exception to this rule (Equation (11)), since its value corresponds to the minimum instantaneous value of the radius of gyration $\rho_p$.

By using these definitions, the nominal translational and rotational periods $T_b$ and $T_{b\theta}$ are defined, respectively, as

$$
T_b = 2\pi \sqrt{\frac{m}{k_b}}, \quad T_{b\theta} = 2\pi \sqrt{\frac{m \rho^2}{k_b \bar{\rho}_p^2}} = T_b \frac{\rho}{\bar{\rho}_p},
$$

and their quotient as the lateral to torsional period ratio (or torsional to lateral frequency ratio) $\Omega_b = T_b / T_{b\theta} = \dot{\rho}_p / \rho$. From Equation (12) it is observed that the nominal radius of gyration $\bar{\rho}_p$. 

or the lateral to torsional period ratio $\Omega_b$, depend on the initial distribution of vertical loads $w_k$ and the distribution in plan of the isolators. By assuming that the isolators are placed on a rectangular grid as the one shown in Figure 1(c), and that the vertical loads $w_k$ are proportional to the tributary area of each isolator, it can be shown [10] that the values of $\Omega_b$ are within $\Omega_b = 1(\Omega_b = \frac{\sqrt{3}}{3} = 1.73)$, corresponding to a uniform distribution of isolators in plan, and $\Omega_b = \frac{\sqrt{3}}{3} = 1.73$, corresponding to the case of an isolator in each of the four corners of the plan. For most practical cases $\Omega_b$ is close to one; for instance, for a regular grid of $3 \times 4$ isolators in a rectangular plan with aspect ratio $\gamma = 1.5$, $\Omega_b = 1.14$, while for a grid of $6 \times 8$ isolators, $\Omega_b = 1.02$.

**EQUIVALENT TORSIONAL INPUT**

Next, an alternative procedure to interpret the accidental torsion caused by overturning in the structure is developed. Let us consider the rotational equation of motion of the isolated base (Equation (4))

$$m_b \rho^2 \ddot{\theta} + V_0 + F_0 = 0$$

where $V_0$ is the base torque of the superstructure and $F_0$ is the torque developed by the restoring forces in the isolators. By assuming a rigid superstructure ($\dot{u}(j) = 0$), replacing in Equation (13) the value of $F_0$ from the third equation of Equation (5), and dividing by $m_t \rho^2$, Equation (13) may be written as

$$\ddot{\theta} + 2 \zeta_b \omega_b \dot{\theta} + \omega_b^2 \theta = \frac{\rho^2}{\rho^2} (e_{py} q_x - e_{px} q_y) + \frac{2 \varepsilon_b \omega_b}{\rho^2} (e_{px} \dot{q}_x - e_{mx} \dot{q}_y)$$

where $\omega_b = \sqrt{k_b/m_t}$ and $\omega_b = \sqrt{k_b/m_t}$ are the instantaneous frequencies in translation and rotation of the isolated rigid superstructure; and $\varepsilon_b = c_b/2m_t \omega_b$ and $\varepsilon_b = c_b/2m_t \omega_b$ are the instantaneous frictional damping ratios in translation and rotation, respectively. Multiplying Equation (14) by half of the longest plan dimension, $b$, and using the expression of the moment of inertia of an area of rectangular plan shape, $\rho^2 = (a^2 + b^2)/12$

$$\ddot{\theta} + 2 \zeta_b \omega_b \dot{\theta} + \omega^2 \theta = \Theta_b(t) \chi(\gamma)$$

with

$$\Theta_b(t) = 3 \omega_b^2 (\varepsilon_{py} q_x - \varepsilon_{px} q_y) + 6 \varepsilon_b \omega_b (\varepsilon_{py} \dot{q}_x - \varepsilon_{mx} \dot{q}_y)$$

and

$$\chi(\gamma) = \frac{2\gamma}{1 + \gamma^2}$$

where $\delta_b = q_0 b/2$ is the lateral deformation on the edge associated with the plan rotation; $e_{px} = e_{px}/a$, $e_{py} = e_{py}/a$, $e_{mx} = e_{mx}/a$, and $e_{py} = e_{py}/a$ are the normalized eccentricities of the isolation system; $\Theta_b(t)$ will be denoted hereafter as the equivalent torsional input (ETI); and $\chi(\gamma)$ is a normalized factor that depends only on the plan aspect ratio $\gamma$.

It is apparent from Equation (16) that $\Theta_b(t)$ is a function of the lateral response of the structure and the normalized eccentricities, which, in turn, are dependent on the slenderness of the structure. Also notice that the function $\chi(\gamma)$ is maximum and equal to 1 for $\gamma = 1$, decreasing asymptotically to zero as the plan becomes narrower. As it will be shown later
numerically, this is due to the fact that as the plan dimension \( b \) increases, the input \( \Theta_b(t) \chi(t) \) is controlled by the variation in the geometric factor \( \chi(t) \) since the overturning moments and, hence, the eccentricities remain essentially constant for a constant slenderness of the structure. Consequently, buildings with square plans are expected to have the largest amplifications for the displacements on the edges of the plan due to accidental torsion. This is an interesting property of the FPS system, which can be used effectively to control the torsional amplification of deformations in structures with elongated plans.

In addition, Equation (16) shows that if the structure is subject to a unidirectional input, the ETI will be zero. This is due mathematically to the cross nature of the products that appear in the expression for \( \Theta_b(t) \); physically, such is the case because accidental torsion occurs as a consequence of the eccentricities produced by the overturning caused by an input component orthogonal to the direction of analysis. Therefore, the characteristics of the input that will control accidental torsion are (i) the relative magnitude of the earthquake components; (ii) their phase differences; and (iii) their statistical correlation.

**RESPONSES AND EARTHQUAKE RECORDS CONSIDERED**

The building responses considered in this study are: (i) the normalized instantaneous stiffness eccentricities \( \hat{e}_{px} \) and \( \hat{e}_{py} \); (ii) the normalized instantaneous frictional eccentricities \( \hat{e}_{PSx} \) and \( \hat{e}_{PSy} \); (iii) the instantaneous radii of gyration for the pendular and frictional components normalized with respect to the radius of gyration of the mass of the isolation base, \( \hat{\rho}_p = \rho_p / \rho \) and \( \hat{\rho}_f = \rho_f / \rho \); (iv) the \( X \)-direction deformation of the isolation base at the CM, \( q_x \); (v) the edge deformation due to base rotation only, \( \hat{\delta}_b^{(b)} = q_b b/2 \); (vi) the total edge deformations at the base in the \( X \)-direction, \( d^{(b)}_{x(\pm b/2)} = q_x \pm \hat{\delta}_b^{(b)} \); (vii) the deformation between the roof and base in the \( X \)-direction measured at the CM, \( u_x^{(n)} \); (viii) the edge deformation between the roof and base due to plan rotation only, \( \hat{\delta}_r^{(b)} = u_r^{(n)} b/2 \); (ix) the total roof-to-base edge deformations in the \( X \)-direction, \( d^{(r/b)}_{x(\pm b/2)} = u_x^{(n)} + \hat{\delta}_r^{(b)} \); (x) the interstorey deformations in the \( X \)-direction at the CM, \( v_x^{(j)} = u_x^{(j)} - u_x^{(j-1)} \), and (xi) the interstorey edge deformations in the \( X \)-direction, \( v^{(j)}_{x(\pm b/2)} = v_x^{(j)} \pm (u_x^{(j)} - u_x^{(j-1)}) b/2 \).

To evaluate the effect of accidental torsion at the base and superstructure, two global parameters are defined. These are the ‘type-\( \Xi \)’ parameters, which represent a quotient between the peak response at the edges of the plan due to plan rotation only and translation at the CM. For instance, \( \Xi_{b/2}^{[b]} \) is defined as the percentual ratio between the deformation at the edge of the building due to rotation only and the lateral deformations at the CM, i.e.

\[
\Xi_{b/2}^{[b]} = 100 \times \frac{\max \| \hat{\delta}_b^{(b)} \|}{\max \| q_x \|} \quad \Xi_{b/2}^{[r/b]} = 100 \times \frac{\max \| \hat{\delta}_r^{(b)} \|}{\max \| u_x^{(n)} \|} \quad (\%)
\]

The second class of parameters are named ‘type-\( \Gamma \)’ and represent a quotient between the total peak response at the edges of the building plan \( (y = \pm b/2) \) and the peak responses at the CM \( (y = 0) \). For instance, \( \Gamma_{\pm b/2}^{[b]} \) is computed as the difference between the maximum total base deformation at the edge of the building relative to the base deformation

at the CM, i.e.

\[
\Gamma^{[b]}_{\pm b/2} = 100 \times \frac{\max \| d^{[b]}_{x(\pm b/2)} \| - \max \| q_x \|}{\max \| q_x \|} \quad \text{(18)}
\]

\[
\Gamma^{[r/b]}_{\pm b/2} = 100 \times \frac{\max \| d^{[r/b]}_{x(\pm b/2)} \| - \max \| u^{(n)}_x \|}{\max \| u^{(n)}_x \|} \quad \text{(19)}
\]

Furthermore, the maximum \( \Gamma \)-response will be denoted as \( \Gamma^{[b]}_{b/2} \), i.e.

\[
\Gamma^{[b]}_{b/2} = \max(\Gamma^{[b]}_{+b/2}, \Gamma^{[b]}_{-b/2}), \quad \Gamma^{[r/b]}_{b/2} = \max(\Gamma^{[r/b]}_{+b/2}, \Gamma^{[r/b]}_{-b/2}) \quad \text{(20)}
\]

This latter parameter is also used to evaluate the maximum torsional amplifications of interstorey deformations

\[
\Gamma^{[l]}_{b/2} = \max(\Gamma^{[l]}_{+b/2}, \Gamma^{[l]}_{-b/2}), \quad j = 1 : n \quad \text{(21)}
\]

where

\[
\Gamma^{[l]}_{\pm b/2} = 100\% \times \frac{\max \| v^{(l)}_{x(\pm b/2)} \| - \max \| v^{(l)}_x \|}{\max \| v^{(l)}_x \|} \quad \text{(22)}
\]

Please note that according to their definition, the type-\( \Xi \) parameters are always an upper bound of the type-\( \Gamma \) parameters; the two coincide if and only if the peak translational and rotational responses occur simultaneously.

The building responses presented next were obtained from eight ground motions, six earthquake records, and two synthetic ground motions compatible with a design spectrum for subduction-zone earthquakes [10]. Summarized in Table I are the peak acceleration values for the three components of each record and the peak structural deformations that would undergo a rigid superstructure isolated with FPS subject to these records. The structures have isolators with radii \( R_0 = 100, 155, \) and 225 cm \( (T_b = 2.0, 2.5, \) and 3.0 s), and a constant friction coefficient \( \mu = 0.07 \).

**ANALYSIS OF RESULTS**

In this section, the influence of the different parameters that control accidental torsion due to the variability in the normal isolators forces is evaluated. The parameters considered are the slenderness ratio of the building \( \beta \), the plan aspect ratio \( \gamma \), and the torsional to lateral frequency ratio \( \Omega_\phi \). In the analysis, lateral and torsional response histories at the isolation level and superstructure are considered. Although the simultaneous action of lateral and torsional motions must be considered in evaluating the total accidental torsion effects in a structure, in some cases the influence of the different parameters is presented in terms of the translational and torsional responses independently, leaving the combination of both motions for the final accidental torsion amplification spectra presented.
Table I. Recorded motions considered.

<table>
<thead>
<tr>
<th>Record</th>
<th>Event</th>
<th>PGA (cm/s²)</th>
<th>Peak X-direction deformation (cm)*</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>X-dir.</td>
<td>Y-dir.</td>
</tr>
<tr>
<td>Newhall</td>
<td>Northridge</td>
<td>578.8</td>
<td>568.9</td>
</tr>
<tr>
<td>(EEUU, 1994)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sylmar</td>
<td>Northridge</td>
<td>824.0</td>
<td>588.6</td>
</tr>
<tr>
<td>(EEUU, 1994)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lucerne Valley</td>
<td>Landers</td>
<td>716.7</td>
<td>790.1</td>
</tr>
<tr>
<td>(EEUU, 1992)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>JMA Kobe</td>
<td></td>
<td>817.8</td>
<td>617.1</td>
</tr>
<tr>
<td>(Japón, 1995)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2 × El Centro</td>
<td>Imperial Valley</td>
<td>2 × 306.9</td>
<td>2 × 210.7</td>
</tr>
<tr>
<td>(EEUU, 1940)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2 × Melipilla</td>
<td>Melipilla</td>
<td>2 × 518.0</td>
<td>2 × 673.0</td>
</tr>
<tr>
<td>(Chile, 1985)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Artificial #1†</td>
<td></td>
<td>657.71</td>
<td>600.7</td>
</tr>
<tr>
<td>Artificial #2†</td>
<td></td>
<td>634.9</td>
<td>623.7</td>
</tr>
</tbody>
</table>

* Response obtained for a rigid body on FPS isolators subject to the corresponding ground motion (µ = 0.07).
† Spectrum compatible records for the Chilean Seismic Code and stiff soil.

**Building slenderness β**

Intuitively, the slenderness of the building β must be related to the importance of the overturning effects in the structure and, hence, should influence the distribution of the isolator normal loads in plan. To start, let us consider the response of six-storey buildings with β = 0.75, 1.5 and 3 subject to the three-component Newhall record. The buildings considered all have the same plan aspect ratio γ = 1, and fixed-base parameters Ts = 0.5 s, Ωs = 1, and ζs = 0.05. The structures use 16 FPS isolators with a nominal radius of curvature R₀ = 155 cm (Tᵣ = 2.5 s) and friction coefficient µ = 0.07, arranged in a regular px × py = 4 × 4 grid (Ωb = 1.10).

Shown in the first row of plots of Figure 2 is the location of the Instantaneous Center of Stiffness (ICS) of the isolation system, defined by the trajectories in plan of the pendular eccentricities e.px and e.py. The second row of plots shows the normalized response histories of the stiffness (pendular) and frictional eccentricities in the X-direction, e.px and e.µx, while the third row of plots shows the normalized pendular and frictional radii of gyration pₓ and p.µ, respectively. Several interesting observations may be drawn from this figure. First, as the slenderness of the structure increases, the ICS moves farther away from the geometric centre.
Figure 2. Response of three six-storey buildings \((\gamma = 1, \ T_s = 0.5 \text{ s}, \ \Omega_s = 1, \ \xi_s = 0.05)\) with different slenderness \(\beta = 0.75, 1.5 \text{ and } 3\), and 16 FPS \((p_x \times p_y = 4 \times 4, \ R_0 = 155 \text{ cm}, \ \mu = 0.07)\), subjected to the Newhall record. Instantaneous Center of Stiffness (top); histories of the normalized pendular and frictional eccentricities (middle); and histories of the normalized pendular and frictional radii of gyration (bottom).
of the plan, reaching the perimeter in some cases for \( \beta = 3 \). This latter result implies that at some instants the building underwent uplift and was supported on a single line of isolators due to the overturning in the structure. Second, it is observed from the second row of plots that the stiffness and frictional eccentricities have very similar traces, which implies that both are affected essentially in the same way by the variation in the normal isolator loads due to the overturning of the structure. For practical purposes, it is shown that the ICS and the Instantaneous Center of Frictional Resistance (ICFR) are coincident. Third, notice from the third row of plots that as long as the structure has a small value of \( \beta \) and there is no uplift in the structure, the radii of gyration of the structure remains essentially constant. This result implies according to Equation (10) that the increase in normal isolator loads in one side of the plan is counteracted by a similar decrease in the opposite side, leaving the value of \( \tilde{\rho}_n \) and \( \tilde{\rho}_n \) essentially constant. In contrast, when uplift occurs such compensation in normal isolator loads is not possible, leading to significant changes in the values of the radii of gyration (Figure 2).

The earthquake response of these six-storey structures with slenderness \( \beta = 1.5, 3, \) and 4.5 and subject to the Newhall and Sylmar records are shown in Figure 3. Translational and rotational response histories for the base \( q_x \) and \( \delta_\theta^{(b)} \) are presented in the left column of plots, and translational and rotational deformation histories of the roof relative to the base, \( u_r^{(6)} \) and \( \delta_\theta^{(r)} \) are shown in the right column of plots. As it should, the base and roof translational displacements and deformations are not significantly affected by the slenderness of the structure, especially for the Newhall record. However, plan rotations are affected by the uneven distribution of normal isolators loads as a result of the large eccentricities caused by the larger overturning for larger values of \( \beta \). Notice that there is a notorious variation in the response of the system for a certain value of \( \beta \) for which the ICS consistently reaches the edge of the plan (imminent overturning); it can be shown that the value \( \beta = 2.25 \) for the Sylmar record is an example of such limit value. This limiting value will be denoted hereafter as \( \beta_{lim} \) and occurs when the building uplifts and is supported by a single line of isolators instantaneously, becoming a mechanism. Such behaviour is represented by a softening of the structure, which implies a lengthening of its fundamental period, and a reduction of the peak translational motions at the base. Although for \( \beta_{lim} \) the deformations of the superstructure tend to decrease in general \([12, 13]\), the impact of the structure with the sliding isolator surface after uplift produces an instantaneous sticking of the building that may lead in some cases to larger interstorey deformations in the superstructure \([6]\) (Figure 3).

Therefore, unless \( \beta > \beta_{lim} \), and in spite of the large values of the instantaneous stiffness and frictional eccentricities for smaller \( \beta \) (Figure 2), accidental torsion at the base do not lead to a significant increase in torsional deformations along the edge of the plan relative to the deformations at the CM. Be aware that such eccentricities would lead to a large increase in edge deformations if they were erroneously considered as static eccentricities in the analysis. On the other hand, accidental torsion leads to a significant increase in edge deformations relative to the deformations at the CM of the superstructure. For values of \( \beta \approx \beta_{lim} \), the deformations along the edge due to translation and plan rotation are of similar magnitude. Therefore, it is important for the design of the structure to stay below the value of \( \beta_{lim} \) for a given earthquake.
Figure 3. Lateral and torsional deformations for six-storey buildings (γ = 1, \( T_s = 0.5 \) s, \( \Omega_s = 1 \), \( \zeta_s = 0.05 \)) with different slenderness ratios \( \beta = 1.5, 3 \) and 4.5, and 16 FPS isolators (\( p_x \times p_y = 4 \times 4 \), \( R_0 = 155 \) cm, \( \mu = 0.07 \)), subject to the Newhall and Sylmar records.

**Plan aspect ratio \( \gamma \)**

The effect of the plan aspect ratio can be easily understood by considering the following building examples. To illustrate the effect of \( \gamma \), let us consider again six-storey buildings with three different plan aspect ratios \( \gamma = 1, 4, \) and 8 subject to the Newhall record. According to the building plan geometry associated with these values of \( \gamma \), the 36 identical isolators in these

structures were arranged in a $6 \times 6$, $4 \times 9$, and $3 \times 12$ grid, respectively. As before, the isolators considered had a radius $R_0 = 155$ cm ($\tilde{T}_s = 2.5$ s) and a constant friction coefficient $\mu = 0.07$. Structures with fixed-base parameters, $T_s = 0.5$ s, $\Omega_s = 1$ and $\zeta_s = 0.05$, and two slenderness ratios, $\beta = 0.75$ and $1.5$, were considered in this case.

The earthquake responses for these structures are summarized in Figure 4. The first and second row of plots shows the torsional deformations of the roof relative to the base $\delta^{(r/b)}_0$ and the base $\delta^{(b)}_0$, respectively. The peak responses for each case are indicated in the right corner of each plot. Furthermore, the last row of plots shows the equivalent torsional input ETI defined earlier. It is apparent from the results that the torsional responses of the buildings $\delta^{(r/b)}_0$ and $\delta^{(b)}_0$ are essentially in-phase and, more important, decrease as $\gamma$ increases. This result is consistent with the theoretical observation stated earlier in relation to the torsional input in Equation (15). Besides, the amplitudes of these rotations are proportional to the slenderness ratio of the building $\beta$ and the function $\chi(\gamma)$ (Equation (16)). Since $\chi(\gamma)$ decreases for increasing $\gamma$, this factor is considered as a significant attenuation factor of accidental torsion. Consequently, the value $\gamma = 1$ will be selected from now on in the study since it provides an upper bound for the effects of accidental torsion.

Finally, the histories of the torsional input $\Theta(t)$ presented show that their values are proportional to the slenderness ratio $\beta$ through the base eccentricities and is essentially independent of the plan aspect ratio $\gamma$ (Equation (16)). This is consistent again with the theoretical analysis performed earlier in relation to the torsional input.

**Uncoupled torsional to lateral frequency ratio of the superstructure $\Omega_s$**

The effect of the torsional to lateral frequency ratio $\Omega_s$ is illustrated in Figure 5 for two six-storey buildings subject to the Newhall record, one of them torsionally flexible with $\Omega_s = 0.85$ (left), and the other, torsionally stiff with $\Omega_s = 1.25$ (right). Otherwise, the structures considered are identical to the ones selected in the previous section for the configuration with $\beta = 1.5$ and $\gamma = 1$. From bottom to top, this figure shows the interstorey deformation histories at the CM ($y = 0$) and edges ($y = \pm b/2$) of the base, 2nd, 4th and 6th levels. Included at the right corner of each plot is the torsional amplification computed for each edge of the plan (Equations (19) and (20)). It is observed that $\Omega_s$ does not influence the deformations at the base, leading to negligible torsional amplifications $T^{(b)}_{b/2}$. However, as stated earlier, the increase in edge deformations in the superstructure is naturally affected by the value of $\Omega_s$. Such increase may reach values such as 30% for torsionally flexible structures. As it should be, the increase in the superstructure becomes smaller, say, less than 5%, for values of $\Omega_s \geq 1.25$.

Another interesting observation is that the torsional amplification of the inter-storey deformations varies with height due to the higher mode contributions. This variation is larger for smaller $\Omega_s$ and becomes less important for torsionally rigid superstructures. Higher mode contributions are particularly significant in structures isolated with the FPS due to: (i) the sudden changes in the direction of the frictional forces, equivalent to introducing small force impulses in the structure; (ii) the higher frequencies that participate in the response as a result of the vertical input and the lateral-vertical coupling caused by the FPS; and (iii) the stick-slip phenomenon of frictional isolators. Thus, a larger torsional stiffness in the superstructure is advantageous in order to reduce the effect of the higher modes.
Figure 4. Torsional response of the roof relative to the base (top), of the base (middle), and equivalent torsional input $\Theta(t)$ (bottom), for six-storey buildings ($\beta=0.75$ and $1.5$, $\gamma=1, 4, 8$, and $\delta=36$ FPS isolators ($P \times \mu=6 \times 6, 4 \times 9$, and $3 \times 12, R_0=155$ cm, $\mu=0.07$), subject to the Newhall record.
Figure 5. Interstorey and base deformations at the CM and edges of the plan for six-storey buildings ($\gamma = 1$, $\beta = 1.5$, $T_s = 0.5$ s, $\Omega_s = 0.85$ and 1.25, $\zeta_s = 0.05$) with 36 FPS isolators ($p_x \times p_y = 6 \times 6$, $R_0 = 155$ cm, $\mu = 0.07$), subject to the Newhall record.
Translational and torsional response spectra

Next, independent spectra of the maximum translational and torsional response amplifications due to overturning of the structure are presented. These spectra are shown as a function of the torsional to lateral frequency ratio $\Omega_t$ (abscissa) and slenderness $\beta$ (ordinates) of the structure. A group of 368 six-storey buildings was analysed with $\Omega_t$ varying from 0.7 to 1.8 in increments of 0.05, and the parameter $\beta$ varied from 0.25 to 4 in increments of 0.25. Otherwise, the buildings all have identical fixed-base parameters $\gamma = 1$, $T_s = 0.5\,s$, and $\xi_s = 0.05$, and an isolation system with 36 identical FPS isolators of radius $R_0 = 155\,\text{cm}$ and arranged in a $6 \times 6$ grid.

The response spectra for the Newhall and Sylmar records are presented in Figure 6 for the base deformation at the CM, $q_{s0}$ (top-left), the deformation of the roof relative to the base at the CM, $u^{(6)}_{s0}$ (bottom-left), the torsional deformation at the base, $\delta^{(b)}_{00}$ (top-right), and the torsional deformation of the roof relative to the base, $\delta^{(r/b)}_{00}$ (bottom-right). To ease the interpretation of the results, the value of $\beta_{\text{lim}}$ is presented in the spectra as a horizontal dashed line. The first observation is that the trends noted earlier in relation to Figure 3 are consistent with the results of this figure, i.e.: (i) the values of $q_{s0}$ and $u^{(6)}_{s0}$ are essentially constant for all $\Omega_t$ as long as $\beta < \beta_{\text{lim}}$, and have a minor tendency to decrease with increasing $\beta$ above this limit value; (ii) the value of $\delta^{(b)}_{00}$ increases with increasing $\beta$, especially with $\beta > \beta_{\text{lim}}$, and is rather independent of $\Omega_t$; and (iii) the value of $\delta^{(r/b)}_{00}$ also increases with $\beta$, but tends to be larger for smaller values of $\Omega_t$. As shown by these two ground motions and the results for different values of $\beta$, accidental torsion due to overturning causes mainly a variation in the rotation of the structure. This increase in rotation with respect to a squat structure is dramatic for $\beta > \beta_{\text{lim}}$, say 20 times, and lead to deformations at the edge of the plan that are usually less than 15% (Figure 6) of the value in translation for the base, but are of comparable magnitude for the deformations of the superstructure.

Torsional amplification spectra

Spectra for the torsional amplification factors (TAF) defined by $\Gamma_{\text{h/2}}$, $\Xi_{\text{h/2}}$, $\Gamma_{\text{r/b}}$, and $\Xi_{\text{r/b}}$ (Equations (17)–(22)) are computed next for the same structures as defined in the previous section, subject to the Lucerne, Sylmar, and Newhall records. Shown in parts (a) and (b) of Figure 7 are the TAF for the base and superstructure, respectively. As shown by the figure, the TAF tend to increase more rapidly for slenderness values $\beta$ above the limiting value $\beta_{\text{lim}}$. The amplification factors observed are considerably larger for the superstructure than for the base. For instance, the true TAF (type-$\Gamma$) at the edge of the base are less than 3% for the records considered, while for the superstructure such amplification may be as large as 70%. However, for $\beta < \beta_{\text{lim}}$ the TAF are usually less than 0.5% and 10% for the base and superstructure, respectively. While the TAF at the base are insensitive to variations in the torsional to lateral frequency ratio of the superstructure ($\Omega_t$); such amplifications are larger in the superstructure for torsionally flexible systems ($\Omega_t < 1$). Furthermore, the rotational TAF (type-$\Xi$) follow similar trends, although their values are usually between 2 and 10 times larger than the type-$\Gamma$ amplification factors. Such difference is due to the simultaneity of the maxima of the plan translation and rotation assumed by the type-$\Xi$ amplification factors, which is not correct. Because of this, only the type-$\Gamma$ amplification factors will be considered hereafter.
Figure 6. Translational and rotational deformations for six-storey buildings (γ = 1, \( T_s = 0.5 \) s, \( \bar{\xi}_s = 0.05 \)) with 36 FPS isolators (\( p_x \times p_y = 6 \times 6 \), \( R_0 = 155 \) cm, \( \mu = 0.07 \)) as a function of \( \Omega_s \) and \( \beta \), subject to the Newhall and Sylmar records: base deformations (top) and roof-to-base deformations (bottom).
Figure 7. Rotational and total amplification spectra for six-storey buildings ($\gamma = 1$, $T_s = 0.5$ s, $\zeta_s = 0.05$) with 36 FPS isolators ($p_x \times p_y = 6 \times 6$, $R_0 = 155$ cm, $\mu = 0.07$) as a function of $\Omega_s$ and $\beta$, subject to the Lucerne Valley, Sylmar, and Newhall records: (a) base, and (b) roof to base.

Finally, the reader may notice that the TAF for the Lucerne valley record are considerably smaller than for the other two records. The reason for this is the strong directionality of this record, which leads to small accidental torsion in the structures [10].
It is interesting to look now at the TAF, $\Gamma^{(\beta)}_{\pm b/2}$, occurring at the different building storeys as shown in Figure 8. Results are presented for three values of the parameter $\beta=1, 1.5$, and $2$ and the four ground motion records indicated. For the records selected, the TAF obtained may be as large as 50% for structures with small torsional to lateral frequency ratio $\Omega_s$, but are usually less than 10% for $\Omega_s>1$ and impulsive ground motions, and 20% for subduction type motions. For values of $\Omega_s<1$, there is a tendency for the upper storeys to show larger TAF (e.g. Newhall). This is probably due to the larger contribution of the higher modes in the response of the upper floors of the structure. In any case, there is an important variability in the values of the TAF for different storeys, which tends to decrease with increasing values.

Figure 8. Torsional amplification spectra for the interstory deformations of six-storey buildings ($\gamma=1$, $T_s=0.5$ s, $\zeta_s=0.05$, $\beta=1, 1.5$ and (2)) with 36 FPS isolators ($p_x \times p_y = 6 \times 6$, $R_0=155$ cm, $\mu=0.07$) as a function of $\Omega_s$, subject to the Newhall, Sylmar, Artificial #1, and $2 \times$ Melipilla records.
of $\Omega_s$. As it should be for a nominally symmetric structure, either edge of the building plan may reach the maximum TAF, depending on the parameters of the system and ground motion considered.

For design purposes, it would be interesting to see if it is possible to estimate the variability of the TAF in height by means of an equivalent isolated single-storey structural model. To illustrate that, the mean and mean-plus-one standard deviation of the TAF for 1, 3, 6 and 9-storey structures, with identical properties as those of the structures of the previous figure and slenderness $\beta = 1.5$, are presented in Figure 9. The first column of plots shows the TAF, $\Gamma^{[r;b]}_{b/2}$, for single-storey systems with three different values of the mass ratio, defined as the quotient between the mass of the isolated base and the total mass of the structure, $x = 1, \frac{1}{3}, \frac{1}{6}$. Analogous results for the 3, 6 and 9-storey structures are presented in the other columns of plots. In this figure, the solid traces represent the mean value of the TAF in height.

$$\Gamma^{[x]}_{b/2} = \frac{1}{n} \sum_{j=1}^{n} \Gamma^{[j]}_{b/2}$$

and in dashed lines, the mean-plus-one and mean-minus-one standard deviation value

$$\hat{\Gamma}^{[x]}_{b/2} = \bar{\Gamma}^{[x]}_{b/2} \pm \frac{1}{\sqrt{n-1}} \sqrt{\sum_{j=1}^{n} (\Gamma^{[j]}_{b/2} - \bar{\Gamma}^{[x]}_{b/2})^2}$$
It is apparent that \( x \) has a strong influence in the response of the structure, and, hence, in the values of the TAF. The TAF for single-storey systems differ considerably from the corresponding TAF for multistorey structures. However, the TAF for the 3, 6 and 9-storey structures maintain a closer similarity. For an equivalent isolated single-storey system one should expect to obtain a good estimation of the mean value of the TAF. Unfortunately, the definition of that equivalent system is not straightforward in this case.

The strong dependency between the TAF and the parameter \( x \) for isolated single-storey systems can be explained intuitively. When \( x \) increases, the structure becomes closer to a condition with fixed base and thus, the modes associated with the deformation of the superstructure have a larger participation in the response.

Shown in Figure 10 are the TAF for six-storey buildings with fixed-base parameters \( T_s = 0.5 \text{s} \) and \( \zeta_s = 0.05 \), plan aspect ratios \( \gamma = 1 \) and 4, supported on 36 FPS (\( p_x \times p_y = 6 \times 6 \) for \( \gamma = 1 \) and 4 \times 9 for \( \gamma = 4 \), \( R_0 = 225 \text{ cm} \), and \( \mu = 0.07 \)), and subject to the Newhall, Artificial #2, and \( 2 \times \text{El Centro} \) records. Part (a) of this figure shows the TAF for the base \( \Gamma_b^{[b]} \) (Equation (20)) and the mean value in height for the TAF, \( \Gamma_s^{[s]} \) (Equation (23)), respectively. The trends observed for the results are consistent with those observed earlier in Figure 4. The TAF decreases with increasing values of \( \gamma \). Indeed, taking the average of the ratio \( \Gamma_b^{[s]}(\gamma = 4)/\Gamma_b^{[b]}(\gamma = 1) \) for the complete range of \( \beta \) and \( \Omega_s \) and for the three ground motions considered, a value of 0.486 is obtained, which is close to the theoretical value of \( \chi(\gamma) = 0.47 \) obtained for \( \gamma = 4 \) (Equation (16)). Furthermore, mean TAF for the superstructure are larger for frequency ratios \( \Omega_s < 1 \), reaching a maximum value for the Newhall record of about 35% for \( \gamma = 1 \) and 15% for \( \gamma = 4 \).

The effect on the TAF of the fixed-base period of the superstructure and isolated period of the complete system is examined in Figures 11 and 12. The mean-plus-one standard deviation of the TAF for the base \( \Gamma_b^{[b]} \) and superstructure \( \Gamma_s^{[s]} \) (Equation (24)) are evaluated for buildings with fixed-base periods \( T_s = 0.3 \), 0.5, and 0.7 s (\( \zeta_s = 0.05 \)), square plan (\( \gamma = 1 \)), and isolated with 36 FPS (\( p_x \times p_y = 6 \times 6 \), \( \mu = 0.07 \)) of radii of curvature \( R_0 = 155 \text{ and } 225 \text{ cm} \) (\( T_b = 2.5 \) and 3.0 s). These TAF have been classified for impulsive (Newhall, Sylmar, JMA, and Lucerne Valley) and non-impulsive ground motions (\( 2 \times \text{El Centro}, 2 \times \text{Melipilla}, \text{Artificial} \#1, \) and Artificial #2). Several observations may be derived from these figures, which summarized this research. First, there is no strong dependency of the TAF and the fixed-base period of the superstructure for the range of values considered in the analysis. Second, by analysing the results, a reasonable design value for the TAF of the base, \( \Gamma_b^{[b]} \), could be taken as 5% for all ground motions and building parameters. Third, the TAF of the interstorey deformations of the superstructure, \( \Gamma_s^{[s]} \), are insensitive to \( T_s \) and increase with increasing slenderness \( \beta \) and decreasing frequency ratio \( \Omega_s \). The maximum values are similar for both types of earthquakes and reach about 50%. However, for structures with \( \Omega_s \geq 1 \) and \( \beta \leq 2 \), such increase is usually less than 20%.

CONCLUSIONS

Accidental torsion in structures isolated with the FPS caused by the variability in the normal isolators loads due to the overturning effect of the superstructure is particularly sensitive to the slenderness of the structure \( \beta \) and plan aspect ratio \( \gamma \). Indeed, the larger torsional
Figure 10. Torsional amplification spectra as a function of $\Omega_s$ and $\beta$, for six-storey buildings ($T_s=0.5$ s, $\xi_s=0.05$) with different plan aspect ratio ($\gamma=1$, and 4) and 36 FPS isolators ($p_x \times p_y=6 \times 6$ and $4 \times 9$, $R_0=225$ cm, $\mu=0.07$): (a) maximum base deformation, and (b) average in height of interstory deformations.
NOMINALLY SYMMETRIC STRUCTURES ISOLATED WITH THE FPS

Figure 11. Mean-plus-one standard deviation of the torsional amplification of the base, $T_{b/2}^{[6]}$, of six-storey buildings ($\gamma=1$, $T_s=0.3$, 0.5, and 0.7 s, $\xi_s=0.05$) with 36 FPS isolators ($p_x \times p_y = 6 \times 6$, $T_b=2.5$ and 3 s, $\mu=0.07$) as a function of $\Omega_s$ and $\beta$, and subject to: (a) impulsive ground motion, and (b) non-impulsive ground motion.

amplifications of the deformation at the edge of the buildings occur for values of $\beta$ exceeding a limiting value denoted as $\beta_{\text{lim}}$ and for a square plan ($\gamma=1$). On the other hand, the torsional amplification factors at the base are essentially insensitive to the torsional to lateral frequency
Figure 12. Mean-plus-one standard deviation of the torsional amplification of the superstructure, $\hat{\Gamma}_{\beta}^{[\gamma]}$, of six-storey buildings ($\gamma = 1$, $T_s = 0.3$, 0.5, and 0.7 s, $\zeta_s = 0.05$) with 36 FPS isolators ($p_x \times p_y = 6 \times 6$, $T_b = 2.5$ and 3s, $\mu = 0.07$) as a function of $\Omega_s$ and $\beta$, and subject to: (a) impulsive ground motion, and (b) non-impulsive ground motion.
NOMINALLY SYMMETRIC STRUCTURES ISOLATED WITH THE FPS

ratio of the superstructure $\Omega_s$, fixed-base period of the superstructure $T_s$, and nominal isolation period $T_b$.

An upper bound of the mean-plus-one standard deviation of the torsional amplification at the isolated base is 5%. This value is conservative for design since it decreases for rectangular plans and the usual slenderness ratios of buildings. Consequently, the FPS is capable of limiting the accidental torsion amplification at the base to values smaller than this, in spite of the potential uplift and impact of the structure.

In contrast, accidental torsion may lead to significant torsional amplifications of the interstorey deformations of the superstructure. As before such amplification increases with larger $\beta$, decreases with larger plan aspect ratios $\gamma$, and decreases with larger $\Omega_s$. Although there exists some influence, these amplifications are not strongly dependent on the fundamental fixed-base period $T_s$ and isolated period $T_b$. An upper bound for the mean-plus-one standard deviation of the torsional amplification for the interstorey deformations is, say, 50%. However, for most practical structures isolated with the FPS with $\Omega_s \approx 1$ and $\beta \leq 2$, such increase in interstorey deformation is usually less than 20%. This value includes the large variation that occurs in the torsional amplification in height, probably due to the influence of higher modes.

An important aspect in the design of structures with the FPS is to estimate the limit slenderness $\beta_{lim}$ associated with the imminent overturning of the structure. If the displacements of the structure due the ground motion considered are below this limit, uplift and impact in the isolators will not cause serious modifications in the lateral–torsional response of the system.

As opposed to the intuitive idea that elongated plans might have a serious problem with accidental torsion due to overturning, structures isolated with the FPS are capable of controlling such torsion due to the attenuating factor $\chi(\gamma)$ presented herein. Therefore, according to the results presented, buildings with rectangular plans are in better condition than buildings with square plan to limit accidental torsion effects due to overturning.

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APPENDIX A

More details of the coupled equations of motion of the structures considered in this study (Figure 1) are presented herein. For future use, the most complete case of structures with plan asymmetry is considered. In order to improve the computational efficiency, the number of dynamic degrees of freedom $u$ of the superstructure is reduced by using the transformation of co-ordinates

$$u = \Phi \eta$$  \hspace{1cm} (A1)

where $\Phi_{\{3n \times n_m\}}$ is a matrix with the $n_m$ modes of the superstructure that are preserved in the analysis, normalized with respect to the mass matrix $M_s$; and $\eta_{\{n_m \times 1\}}$ is the vector of natural co-ordinates.
By using Equation (A1) and some algebra, Equations (2)–(4) may be written as the system of coupled equations

\[
\begin{bmatrix}
\ddot{m}_r & \ddot{m}_{rq} & \ddot{m}_{rq} \\
\ddot{m}_{rq} & \ddot{m}_q & \ddot{m}_{qq} \\
\end{bmatrix}
\begin{bmatrix}
\ddot{r} \\
\ddot{q} \\
\end{bmatrix}
+ \begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
\end{bmatrix}
\begin{bmatrix}
\dot{r} \\
\dot{q} \\
\end{bmatrix}
+ \begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
\end{bmatrix}
\begin{bmatrix}
r \\
q \\
\end{bmatrix}
+ \begin{bmatrix}
F_r \\
F_q \\
\end{bmatrix}
= \begin{bmatrix}
B_{wr} \\
B_{wq} \\
\end{bmatrix} w
\]

where

\[
\ddot{m}_r = \begin{bmatrix}
m_1 & -S_{YZ} & S_{XZ} \\
-S_{YZ} & I_{YY} & -I_{XY} \\
S_{XZ} & -I_{XY} & I_{XX} \\
\end{bmatrix}
\]

\[
\ddot{m}_{rq} = H^T M_r \Phi \\
\ddot{m}_{rq} = H^T M_q \Phi \\
\ddot{m}_q = m_0 + R^T M_s R \\
\ddot{m}_{qq} = R^T M_s \Phi \\
\]

\[
B_{wr} = -\begin{bmatrix}
S_{XY} & 0 & -S_{YZ} \\
0 & -S_{XY} & S_{XZ} \\
\end{bmatrix}
\]

\[
B_{wq} = -m_0 L_w^{(0)} \\
B_{wq} = -\Phi^T M_s L_w^{(f)}
\]

where the symbol \(\ldots\) denotes a diagonal matrix; \(\mathbf{I}\) is the identity matrix and \(\mathbf{0}\) is the zero matrix; \(\omega_i\) and \(\zeta_i\) are frequency and damping ratio corresponding to the \(i\)th mode \(\Phi(:,i)\) of the structure with fixed base; \(\mathbf{R}_{(3n \times 3)} = \begin{bmatrix} \mathbf{I} & \mathbf{I} & \ldots & \mathbf{I} \end{bmatrix}^T\) is the matrix that represents the rigid-body motion in translation and rotation of the floor slabs; \(\mathbf{H}_{(3n \times 3)} = [\mathbf{H}^{(1)}; \mathbf{H}^{(2)}; \ldots; \mathbf{H}^{(n)}]\) is the overturning matrix of the superstructure; \(I_{YY} = \sum_{j=0}^n I_{Yj} + m_j (h_j^2 + e_{Xj}^{(f)^2})\) and \(I_{XX} = \sum_{j=0}^n I_{Xj} + m_j (h_j^2 + e_{Xj}^{(f)^2})\) are the inertias of the system about the \(Y\)- and \(X\)-axes, respectively; \(I_{XY} = \sum_{j=0}^n m_j e_{Xj}^{(f)} e_{Xj}^{(f)}\) is the cross-inertia with respect to those axes; \(S_{XY} = \sum_{j=0}^n m_j h_j\), \(S_{YZ} = \sum_{j=0}^n m_j e_{Yj}^{(f)}\), and \(S_{XZ} = \sum_{j=0}^n m_j e_{Xj}^{(f)}\) are the first-order moments with respect to the Cartesian planes; and \(\mathbf{F}_r\) and \(\mathbf{F}_q\) are the non-linear restoring force vectors of the isolators, collocated on the degrees of freedom \(r\) and \(q\), respectively, i.e.

\[
\mathbf{F}_r = \mathbf{L}_r^T \mathbf{f}_z \quad \text{and} \quad \mathbf{F}_q = \mathbf{L}_q^T \mathbf{f}_h
\]
where $f_z[p \times 1] = [f_{z_1}, f_{z_2}, \ldots, f_{z_p}]^T$ contains the vertical forces $f_{z_k}$ of the FPS; $f_h[p \times 1] = [f_{h_1}^T, f_{h_2}^T, \ldots, f_{h_p}^T]^T$ has the horizontal forces of the FPS, $L_{hl}^T = [f_{x_1}, f_{x_2}]^T$; $L_r[p \times 3] = [L_{r_1} ; \ldots ; L_{r_p}]$ and $L_q[2p \times 3] = [L_{q_1} ; \ldots ; L_{q_2} ; \ldots ; L_{q_p}]$ are the kinematic transformation matrices between the local degrees of freedom of the FPS and $r$ and $q$, respectively; and $L_{r_1}(1 \times 3) = [1 - x_k, y_k]$ and $L_{q_2}(2 \times 6) = [1 0 - y_k ; 0 1 x_k]$ only depend on the location of the isolator. Since the FPS isolators are placed in downward position, the $P - \Delta$ effects are transmitted to the foundation and not to the superstructure [7, 8]. This is why the matrices $L_r$ and $L_q$ are constant and do not contain non-linear terms that depend on the response [7, 8].

**APPENDIX B**

To compute the restoring force $F_q$ acting on the isolation base, we proceed as follows:

(i) compute the horizontal deformations of the isolators $\delta_k = [\delta_{x_k}, \delta_{y_k}]^T$ and its deformation velocities $\dot{\delta}_k = [\dot{\delta}_{x_k}, \dot{\delta}_{y_k}]^T$ by using the linear kinematic transformation imposed by the isolation base

$$\begin{align*}
\delta_{h_k} &= L_{q_k} \hat{q} = [q_x - q_y y_k, q_y + q_y x_k]^T \\
\dot{\delta}_{h_k} &= L_{q_k} \hat{\dot{q}} = [\dot{q}_x - q_y \dot{y}_k, \dot{q}_y + q_y \dot{x}_k]^T
\end{align*} \tag{B1}$$

(ii) compute the vertical motion of the isolators by imposing the non-linear kinematic constraint imposed by the spherical sliding surface

$$\delta_{z_k} = R_k - \sqrt{R_k^2 - (\delta_{x_k}^2 + \delta_{y_k}^2)} , \quad \dot{\delta}_{z_k} = \frac{\dot{\delta}_{x_k} \delta_{x_k} + \dot{\delta}_{y_k} \delta_{y_k}}{(R_k - \delta_{z_k})} \tag{B2}$$

(iii) compute the normal $\hat{n}_k$ and tangent $\hat{t}_k$ vector for each isolator,

$$\begin{align*}
\hat{n}_k &= \frac{1}{R_k} [\delta_{x_k}, \delta_{y_k}, \delta_{z_k} - R_k]^T , \\
\hat{t}_k &= \frac{\hat{\delta}_k}{\|\hat{\delta}_k\|}
\end{align*} \tag{B3}$$

(iv) evaluate the isolator forces $f_k$ using the constitutive relationship [7, 8]:

$$f_k = [f_{h_k}; f_{z_k}] = f_{k}^{(p)} + f_{k}^{(\mu)} = N_k \hat{n}_k + \mu_k N_k \hat{t}_k \tag{B4}$$

where $f_{k}^{(p)}$ and $f_{k}^{(\mu)}$ are the pendular and frictional components of the force, respectively; and $N_k$ is the magnitude of the normal isolator force.

(v) impose the equilibrium conditions and use Equations (B1) and (B4), to obtain

$$F_q = L_{hl}^T f_h = \sum_{k=1}^{p} L_{q_k}^T (f_{h_k}^{(p)} + f_{h_k}^{(\mu)}) = \sum_{k=1}^{p} N_k \frac{L_{q_k}^T \delta_{h_k}}{R_k} + \sum_{k=1}^{p} \frac{\mu_k N_k}{\|\hat{\delta}_k\|} L_{q_k}^T \hat{\delta}_{h_k}
= \left( \sum_{k=1}^{p} \frac{N_k}{R_k} L_{q_k}^T L_{q_k} \right) \hat{q} + \left( \sum_{k=1}^{p} \frac{\mu_k N_k}{\|\hat{\delta}_k\|} L_{q_k}^T L_{q_k} \right) \hat{\dot{q}} = K_h \hat{q} + C_h \hat{\dot{q}} \tag{B5}$$

where $K_h$ and $C_h$ may be interpreted as the instantaneous secant stiffness and damping matrices of the isolation system. By developing the summations that define each of these matrices one obtains Equations (6)–(10).
REFERENCES


